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PHASE TRANSITIONS IN EXPERIMENTAL DYNAMICAL SYSTEMS

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Abstract: *The scaling function $\psi(\lambda)$ is calculated for two characteristic experimental situations of an NMR-laser system. The results agree with those obtained by H.Hata et al., indicating that $\psi(\lambda)$ is a powerfull tool to characterize experimental dynamical systems.*

1. Introduction

In the last years it has been realized that the chaotic behavior of dynamical systems is due to nonlinear effects produced by the dynamical equation. A description of these effects involves spatial as well as temporal aspects. While the probabilistic description of effects in the phase space has been reported vastly ($f(\alpha)$, fractal dimensions), not many publications deal with the numerically more involved dynamical description ($\psi(\lambda)$, Lyapunov exponents). By looking at simple examples, relations between the two directions of description can be derived analytically. It is found that they describe fundamentally different aspects of the scaling behavior of a strange attractor and that only in special cases the result of one description can be deduced from the other [1]. The complete scaling behavior of a dynamical system can be formulated in terms of a generalized thermodynamical formalism. In the usual way a generalized free energy and a generalized entropy are derived. By imposing constraints, generalized dimensions, Lyapunov exponents and associated scaling functions can be obtained.

2. Phase-transition-like effects in experimental systems

While for the simplest, hyperbolic, model cases the associated thermodynamic function is analytic, phase-transition-like effects can be observed for more complicated systems. Well known examples include the logistic, Henon's, and the circle map. Situations where these effects take place can be detected easier in the graph of the scaling function (corresponding in the thermodynamic formalism to the entropy) since there they lead to a piece of non-strictly convex behavior. They are of interest, since, as in the usual thermodynamic formalism, the different phases indicate different degrees of order, different dominant structures in the dynamical system can be distinguished and therefore a simpler global characterization of the dynamical system can be given. In this sense, many of these phase-transition-like effects can be explained as the result of a coexistence between different simpler systems. A special effect is provided by the existence of homoclinic tangency points for nonhyperbolic systems, which is believed to be generic for experimental systems.
For experimental systems, the information about the dynamical system is provided by the help of a time series. From this, the attractor is reconstructed by an embedding process and the generalized dimensions and the generalized Lyapunov exponents and the corresponding scaling functions can be calculated. The method used to calculate Lyapunov exponents from time series is a further developed version of the algorithm described in ([3]).

3. Results and discussion

In this contribution, experimental data from an NMR-laser system are examined.

While changing parameters, the system undergoes different metamorphoses, some of them involve fast changes of the form of the associated attractors (so-called crises), e.g. when two coexisting chaotic bands merge due to a collision with an unstable fixed point. In Figure 1 the scaling function for an experimental situation far away from crisis is shown. It can be shown that a three-dimensional system with a dynamical map of a quadratic maximum should have a linear part in the scaling function $-\psi(\lambda)$ of slope one. As is easily seen, this is here the case. In the inset, the scaling function of an experimental situation near a crisis is shown. The traces of two involved attractors are clearly visible. Both figures are in complete agreement with the results of Horita et al.[4], where however the scaling functions were calculated directly for model maps, using the known dynamical equations.

4. References

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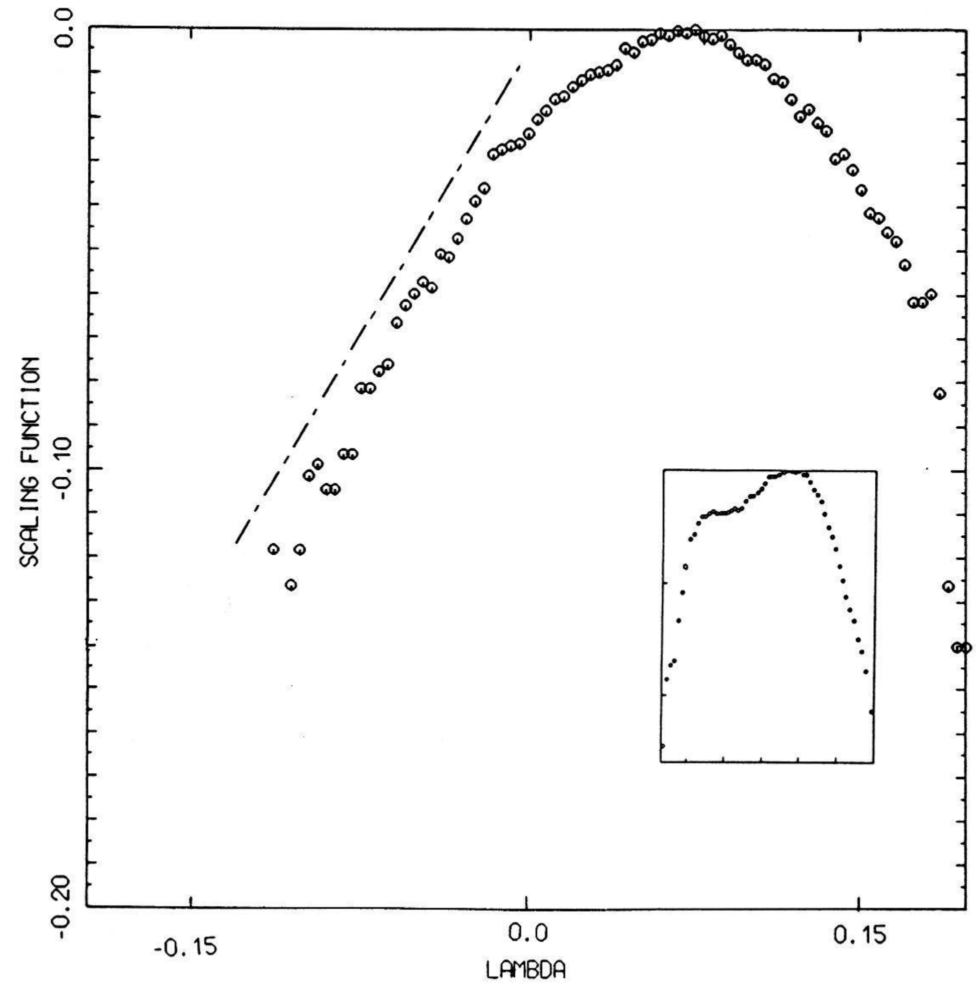

Figure 1: Approximated scaling function $-\psi(\lambda)$ for a file far away from crisis. Inset: file near a crisis. Dashed line: linear region (see text)