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Baryon Matrix Elements of the Vector Current in Chiral Perturbation Theory[1]

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Abstract

The article is concerned with the vector form factors of the weak current of semileptonic decays of hyperons. Our main interest is in the asymmetry of the form factor $F_1(0)$; for a precise determination of the Kobayashi-Maskawa matrix element V_{us} the asymmetry has to be taken into account. In addition we look at the form factor $F_2(0)$, which is related to the baryon magnetic moments. Using chiral perturbation theory we analyse the leading singularities of these form factors.

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Introduction

The information concerning the matrix element V_{us} of the Kobayashi–Maskawa matrix derives from semileptonic decays of K-mesons and of hyperons. The main problem in the determination of V_{us} from the experimental information on these decays stems from the fact that the transition amplitude involves a matrix element of the weak current between hadronic states, which cannot be calculated ab initio with presently known techniques.

In the case of the decays $K \to \pi l \nu$, the axial current does not contribute. The meson matrix element of the vector current involves two form factors $f_+(q^2), f_-(q^2)$. The contribution generated by f_- is proportional to the lepton mass. In the electronic decay channel this contribution is therefore negligibly small. Furthermore, the q^2-dependence of the form factor f_+ can be determined experimentally on the basis of the Dalitz plot distribution. With this information, the data on the decays $K^+ \to \pi^0 e^+ \nu_e$ and $K_L \to \pi^- e^+ \nu_e$ allow one to very accurately determine the quantities $|V_{us} \cdot f_+^{K^+\pi^0}(0)|$ and $|V_{us} \cdot f_+^{K_L\pi^-}(0)|$. To extract the value of $|V_{us}|$ from these quantities, theoretical information concerning the size of the meson form factors at zero momentum transfer is indispensable.

In the theoretical limit in which the quark masses m_u, m_d, m_s are set equal, the vector charge is conserved and the form factor $f_+(0)$ reduces to a Clebsch–Gordan coefficient. The Ademollo–Gatto theorem [1] asserts that the asymmetries in $f_+(0)$ generated by the quark mass differences are small, of order $(m_s - \hat{m})^2$; $\hat{m}$ denotes the mean mass of the up and down quark. As pointed out by Li and Pagels [24], the coefficient of $(m_s - \hat{m})^2$ is however singular in the chiral limit $(m_u, m_d, m_s \to 0)$. Since the physical quark masses are small, the occurrence of a singularity at $m_q = 0$ may imply that the form factor $f_+(0)$ contains sizeable asymmetries. The problem is analysed in detail in [25].

The main subject of the present article is the extension of this analysis to the baryon matrix elements of the weak current. These matrix elements play a crucial role in the extraction of V_{us} from the decays

$$\Sigma^- \to n e^- \bar{\nu}_e \qquad \Xi^- \to \Lambda e^- \bar{\nu}_e$$

$$\Xi^- \to \Sigma^0 e^- \bar{\nu}_e \qquad \Lambda \to p e^- \bar{\nu}_e \; .$$

In these decays, both the vector and axialvector current

$$V_\mu^P = \bar{q} \gamma_\mu \frac{\lambda^P}{2} q$$

$$A_\mu^P = \bar{q}\gamma_\mu\gamma^5\frac{\lambda^P}{2}q\,.$$

contribute to the transition amplitude. Denoting the one-particle states by $|B^P;p>$ where P is an octet label, we have

$$\begin{aligned} < B^P;p|V_R^\mu(0)|B'^Q;p'> &= \bar{u}(p)\left(F_1^{PQR}(q^2)\gamma^\mu + i\frac{F_2^{PQR}(q^2)}{M_P+M_Q}\sigma^{\mu\nu}q_\nu \right.\\ &\quad \left. +\frac{F_3^{PQR}(q^2)}{M_P+M_Q}q^\mu\right)u(p') \end{aligned}$$

$$\begin{aligned} < B^P;p|A_R^\mu(0)|B'^Q;p'> &= \bar{u}(p)\left(G_1^{PQR}(q^2)\gamma^5\gamma^\mu + i\frac{G_2^{PQR}(q^2)}{M_P+M_Q}\gamma^5\sigma^{\mu\nu}q_\nu \right.\\ &\quad \left. +\frac{G_3^{PQR}(q^2)}{M_P+M_Q}\gamma^5 q^\mu\right)u(p') \end{aligned}$$

$$q = p' - p \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,.$$

The form factors $F_3(q^2)$ and $G_3(q^2)$ generate negligibly small contributions proportional to the electron mass. To a very high degree of accuracy the semileptonic hyperon decays are therefore described by the four functions $F_1(q^2)$, $F_2(q^2)$, $G_1(q^2)$, $G_2(q^2)$, rather than by a single function $f_+(q^2)$ as in the mesonic case. Accordingly the analysis of the data is considerably more complex.

Since the theoretical information about the axialvector form factors G_1, G_2 is rather crude, it is doubtful whether it makes sense to use this information in the analysis, aimed at a measurement of V_{us} at the $1\% - 2\%$ level of accuracy. Both the relative magnitude of the form factors and their dependence on q^2 manifest themselves in the angular distribution of the decay. One can therefore exploit the experimental information on this distribution to fix the decay rate in terms of a single constant, which may be identified with the value of the form factor $F_1(q^2)$ at $q^2 = 0$. The data on the various baryon decays therefore allow one to measure the quantities $|V_{us}\cdot F_1^{\Lambda p}(0)|$, $|V_{us}\cdot F_1^{\Sigma^- n}(0)|$ etc. without invoking questionable theoretical models. The essential theoretical information which is needed to determine $|V_{us}|$ is thus the form factor $F_1(0)$. As it is the case with $f_+(0)$, the Ademollo–Gatto theorem suppresses the asymmetries in $F_1(0)$. The singularities which occur in the chiral limit may however enhance the asymmetry substantially. In the following, we analyse the structure of these singularities in detail. We will present our conclusions concerning the numerical size of the asymmetries at the end of this work. The method allows us at the same time to analyse the asymmetries in the form factor $F_2(0)$, related to the baryon magnetic moments.

Our analysis is based on chiral perturbation theory, i. e. on an expansion of the current matrix elements in powers of the momenta and of the quark masses m_u, m_d, m_s. In the chiral limit, the strong interaction is invariant under $SU(3) \times SU(3)$ flavour (chiral) transformations. V_μ and A_μ are the conserved currents of this symmetry. The symmetry is implemented on the particle states in a Nambu-Goldstone realization. The charges of the vector current leave the vacuum invariant; the charges of the axialvector current spontaneously break the symmetry of the vacuum. The corresponding Goldstone bosons are identified with the lightest hadrons — the pseudoscalar mesons. In the real world the masses of the light quarks do not vanish, but are small. The currents are not conserved; instead one finds e.g.

$$\partial_\mu(\bar{u}\gamma^\mu s) = i(m_u - m_s)\bar{u}s$$

$$\partial_\mu(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d)\bar{u}\gamma^5 d\,.$$

One is thus dealing with an approximate symmetry of the strong interaction with the quark masses as the symmetry breaking parameters.

The first ideas how to deal with an approximate symmetry date back to Gell-Mann [18]. Later Fubini et al. [13] introduced the current algebra, which assumes that at equal times the currents form elements of the Lie algebra of the symmetry group. Together with PCAC (partially conserved axialvector current) it was used to calculate current matrix elements. A different approach has been developed by Glashow and Weinberg [19], who derived exact Ward identities for the Green's functions and then used pole dominance to obtain quantitative results. In both methods it is difficult to control the approximations. Dashen and Weinstein [11,12] pointed out that these two approaches are equivalent to a perturbative expansion in the small symmetry breaking parameter - the quark mass (chiral perturbation theory). Since the theory contains massless Goldstone bosons in the symmetric limit, the expansion in the quark mass involves nonanalytic pieces as was shown by Li and Pagels [24]. A consistent framework for chiral perturbation theory was given by Langacker and Pagels [22]. As a reference for further details we recommend the excellent review article of Pagels [28].

Weinberg [34] recognized that chiral perturbation theory is equivalent to the perturbation expansion of an effective chiral Lagrangian theory. Using the effective Lagangian approach, Gasser and Leutwyler [15,16,17] coupled the quark currents to external fields. This framework is the most convenient one to calculate Green's functions and S-matrix elements. It has been successfully applied to various meson matrix elements and recently also to $\pi - N$ scattering. Here we use this technique to investigate the leading corrections of the symmetric limit for the form factors F_1 and F_2 associated with the baryon matrix elements of the vector current. The calculation has been worked out up to one loop in an effective baryon meson theory.

The organization of this article is as follows. In the first chapter we relate the decay rate and the magnetic moment to the form factors. A description of the framework and a construction of the effective baryon meson Lagrangian is given in the following two chapters. We are concerned with the analytic expressions of the one loop diagrams in chapter 4 using the effective Lagrangian to lowest order. The next chapter is devoted to the wave function and mass renormalization of the baryons. In chapter 6 we investigate the combinatorics of the contributions to a given physical process. In the last chapter we use approximations of the analytic expressions of the one loop diagrams in a numerical analysis. We finish the work with the conlusions. Appendix A is devoted to the conventions and notations used in this work. The last two appendices contain the list of counterterms of the effective Lagrangian and the analytic expressions of the one loop diagrams.

Chapter 1

Decay Rates and Magnetic Moments

1.1 Decay rates of semileptonic processes

We consider semileptonic processes of the form

$$B \rightarrow B' + e + \bar{\nu}_e,$$

where B, B' denote baryons, e the electron and $\bar{\nu}_e$ the electron-antineutrino. The specific decays we are interested in are listed in the introduction. With the normalization condition of fermion states of appendix A, the decay rate Γ for such a process is given by [3,27]

$$\Gamma = \frac{1}{2E_B} \int d\mu(p')\, d\mu(p_e)\, d\mu(p_\nu)(2\pi)^4 \delta^4(p - p' - p_e - p_\nu) \frac{1}{2} \sum_{spins} |T|^2 ,$$

where

$$d\mu(p) = \frac{d^3p}{(2\pi)^3(2E_p)} .$$

Since the momentum transfer q in these processes is small compared to M_W, the weak interaction part of the Standard model reduces to the Fermi theory [10,26]. The radiative corrections due to the electromagnetic interaction can be treated perturbatively [31,30]. In the absence of radiative corrections the S-matrix element T factorizes into a leptonic and a hadronic matrix element

$$T = \frac{G_F}{\sqrt{2}} < B|H_\mu|B' > \cdot < l|L^\mu|\nu_l >$$

with

$$L_\mu = \bar{e}\gamma_\mu(1 - \gamma^5)\nu_e$$

$$H_\mu = V_{ud}\bar{u}\gamma_\mu(1 - \gamma^5)d + V_{us}\bar{u}\gamma_\mu(1 - \gamma^5)s.$$

Here G_F is the Fermi constant and V_{ud}, V_{us} denote the elements of the Kobayashi-Maskawa matrix. The leptonic matrix element is known explicitly

$$< l|L^\mu|\nu_l >= \bar{u}_e(p_e)\gamma_\mu(1-\gamma^5)v_\nu(p_\nu)\,.$$

The hadronic matrix element is parametrized by the form factors F_i and G_i introduced in the introduction.

1.2 Magnetic moments

The magnetic moment μ measures the response of a particle to an applied magnetic field. In order to derive a relation between the magnetic moment and the form factors, we look at the baryon matrix elements of the electromagnetic current j_μ. They are parametrized in analogy to the weak current matrix elements

$$\begin{aligned} < B^P;p|j^\mu(0)|B'^Q;p' > &= \bar{u}(p)\left(F_1^{PQ}(q^2)\gamma^\mu + i\frac{F_2^{PQ}(q^2)}{M_P+M_Q}\sigma^{\mu\nu}q_\nu \right. \\ &\quad \left. + \frac{F_3^{PQ}(q^2)}{M_P+M_Q}q^\mu\right)u(p') \end{aligned} \tag{1.1}$$

with

$$j_\mu = \bar{q}\gamma_\mu\frac{1}{2}(\lambda^3+\frac{1}{\sqrt{3}}\lambda^8)q\,. \tag{1.2}$$

A comparison of the nonrelativistic limit of the right hand side with the Pauli equation leads to the desired formula [20]. With the conventions used in equation (1.1) to define the form factors we find

$$\mu^{PQ} = \frac{F_1^{PQ}(0)-F_2^{PQ}(0)}{M_P+M_Q}\,. \tag{1.3}$$

The form factors have a direct physical interpretation, e.g. $F_1(0)$ is the electric charge, $F_1'(0)$ determines the charge radius of the particle and $F_2(0)$ is the anomalous magnetic moment of the baryon.

The electromagnetic current as well as the weak vector and axial vector current are expressed in terms of quark fields. The baryons are bound states of quarks. Until now there exists no prescription of the bound states in terms of quarks within QCD. There is thus no simple way to compute baryon matrix elements. Moreover we are interested in processes with small momentum transfer. Since QCD is an asymptotically free theory, the coupling of the quarks and gluons is strong in the low energy region. A perturbative approach, as it is used e.g. in deep inelastic scattering of nucleons, is therefore not possible.

There exist two fundamentally different approaches to examine the form factors. The first one uses a phenomenological description of the baryons based on the naive quark model [14]. It introduces wave functions for the baryon states in analogy to a description of the hydrogen atom. The free parameters which specify the wave functions have to be fixed by experiment. Having expressed the baryons in term of quarks an evaluation of the baryon matrix elements becomes possible. With this approach one can obtain only rough estimates of the asymmetry of the form factors. It does not have the accuracy needed to determine the Kobayashi–Maskawa matrix element V_{us}.

The other approach exploits only the symmetry properties of the strong interaction. It is *not* based on a phenomenological description and on additional asumptions. As discussed in the introduction an expansion in the quark mass can be set up (chiral perturbation theory). Using this approach we are able to determine the asymmetry of the form factor $F_1(0)$ to a high accuracy. In the next chapter we discuss how one can obtain a low energy representation of the form factors.

Chapter 2

Exploiting Chiral Symmetry

2.1 Vacuum transition amplitude

Before investigating the baryon-baryon transition amplitude, we first look at the vacuum-vacuum transition amplitude in the presence of external fields

$$e^{iZ[v,a,s,p]} =< 0_{out}|0_{in} >_{v,a,s,p} , \tag{2.1}$$

which is based on the Lagrangian

$$\mathcal{L} = \mathcal{L}^0_{QCD}+\bar{q}(x)\gamma^\mu v_\mu(x)q(x)+\bar{q}(x)\gamma^5\gamma^\mu a_\mu(x)q(x)-\bar{q}(x)s(x)q(x)+i\bar{q}(x)\gamma^5 p(x)q(x) . \tag{2.2}$$

$$\mathcal{L}^0_{QCD} = -\frac{1}{2g^2}\mathrm{tr}_c(G_{\mu\nu}(x)G^{\mu\nu}(x)) + \bar{q}(x)i\gamma^\mu(\partial_\mu - iG_\mu(x))q(x) ,$$

where G_μ is the gluon field and $G_{\mu\nu}$ is the corresponding fieldstrength tensor; tr_c denotes the trace over the $SU(3)$ colour group. In the following we restrict ourselves to the $SU(3)$ flavour group. The external fields $v_\mu(x), a_\mu(x), s(x), p(x)$ are coupled to the vector, axialvector, scalar, and pseudoscalar current of the quarks respectively. They are 3×3 hermitean, colour neutral matrices in flavour space. We are only interested in the octet part of the external fields v_μ, a_μ, p and therefore put $\mathrm{tr}(v_\mu) = \mathrm{tr}(v_\mu) = \mathrm{tr}(p) = 0$. The quark mass matrix $\mathcal{M}$

$$\mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

is included in the scalar field $s(x)$. Thus $\mathcal{L}^0_{QCD}$ is the QCD Lagrangian with three massless quark flavours.

The functional Z generates the Green's functions of the quark currents. Their low energy structure has been studied extensively by Gasser and Leutwyler [16,17]. The path integral representation of Z is given by

$$e^{iZ[v,a,s,p]} = \int \mathcal{D}G_\mu \mathcal{D}q\mathcal{D}\bar{q}\; e^{i\int d^4x\, \mathcal{L}(q,\bar{q},G_\mu;v,a,s,p)} . \tag{2.3}$$

The Lagrangian $\mathcal{L}$ is invariant with respect to local $SU(3) \times SU(3)$ flavour transformations (chiral transformations), if the quark fields and the external fields transform as follows:

$$
\begin{aligned}
q_R'(x) &= V_R(x)\, q_R(x) \\
q_L'(x) &= V_L(x)\, q_L(x) \\
v_\mu'(x) + a_\mu'(x) &= V_R(x)\,(v_\mu(x) + a_\mu(x))\, V_R^\dagger(x) + iV_R(x)\partial_\mu V_R^\dagger(x) \\
v_\mu'(x) - a_\mu'(x) &= V_L(x)\,(v_\mu(x) - a_\mu(x))\, V_L^\dagger(x) + iV_L(x)\partial_\mu V_L^\dagger(x) \\
s'(x) + ip'(x) &= V_R\,(s(x) + ip(x))\, V_L^\dagger \,,
\end{aligned}
\tag{2.4}
$$

where $V_R, V_L \in SU(3)$ and where $q_R = \frac{1}{2}(1+\gamma_5)q, q_L = \frac{1}{2}(1-\gamma_5)q$ denotes the right- and left-handed component of the quark field respectively. However the generating functional $Z(v,a,s,p)$ is not invariant under the full group of chiral transformations; the quantization leads to chiral anomalies. The general structure of the anomalies has been given by Bardeen and by Wess and Zumino [2,35]. The generating functional can be split into two pieces

$$
Z[v,a,s,p] = Z_{anom}[v,a,s,p] + Z_{inv}[v,a,s,p] \,,
$$

where the anomalous part Z_{anom} is known explicitly. What has to be done is to find a representation of the invariant part Z_{inv}.

Since the momenta occuring in the physical processes are small, we need to know the Green's functions or the corresponding generating functional only for small momenta compared to the scale of the theory, which is of the order of 1 Gev. Expanding the Green's function in powers of the external momenta amounts to expanding the generating functional Z in powers of the derivatives of the external fields. One might therefore expect Z to be a polynomial in the external fields and derivatives thereof. The low energy expansion of the Green's functions is, however, not a Taylor expansion; chiral symmmetry is spontoneously broken by the vacuum of QCD, the Goldstone bosons being identified with the mesons. The Goldstone bosons generate poles at $q^2 = 0$ in the chiral limit or at $q^2 = M^2_{Meson} = O(\mathcal{M})$, if the quark masses are not exactly zero. This leads to nonlocal terms in Z.

The low energy expansion involves two small parameters: the momentum q and the quark masses $\mathcal{M}$. In order to find the low energy behaviour of the Green's functions one has to expand them both in powers of q and $\mathcal{M}$ for fixed ratio $\mathcal{M}/q^2$. Effective Lagrangians form a convenient method to carry out the expansion for the generating functional coherently [11,12]. One can show that Z has the following low energy representation

$$
e^{iZ[v,a,s,p]} = \int \mathcal{D}U e^{i\int d^4x\, \mathcal{L}_{Mes}(U;v,a,s,p)} \,,
\tag{2.5}
$$

where $\mathcal{L}_{Mes}$ is an effective meson theory. The low energy expansion is then obtainable from a perturbative expansion of the effective meson field theory.

In the effective theory the mesons are collected in a unitary 3×3 matrix U

$$U^\dagger U = \mathbf{1} \qquad \det U = 1 \;, \tag{2.6}$$

which transforms linearly under chiral transformations

$$U' = V_R U V_L^\dagger \qquad V_R, V_L \in SU(3) \;. \tag{2.7}$$

In what follows we parametrize U as follows

$$U = e^{i\Phi} \;, \tag{2.8}$$

where Φ is a traceless, hermitean 3×3 matrix in flavour space(see appendix A). The choice of a linear representation is not the only possible one, however, most convenient.

2.2 Baryon transition amplitude

In order to analyze the low energy structure of the baryon form factors, we extent the effective Lagrangrian to

$$\mathcal{L} = \mathcal{L}_{Mes} + \mathcal{L}_{HM}(B, \bar{B}, U; v, a, s, p) \;. \tag{2.9}$$

$\mathcal{L}_{Mes}$ is the effective meson Lagrangian; it has already been constructed by Gasser and Leutwyler [16,17]. $\mathcal{L}_{HM}$ describes the effective baryon-meson interaction. We collect the baryons in a 3×3 traceless hermitean matrix B, transforming nonlinearly under chiral transformations (see appendix A) [9,33,4]

$$B' = RBR^\dagger \;. \tag{2.10}$$

$R(V_L, V_R, U)$ is a nonlinear function of the meson field U and V_L, V_R, defined by

$$u' = V_R u R^\dagger = R u V_L^\dagger \;, \tag{2.11}$$

where

$$U = u^2 \qquad U' = u'^2 = V_R U V_L^\dagger \;.$$

The baryon field can be expanded in terms of the Gell–Mann matrices λ^a

$$B = \frac{1}{\sqrt{2}} B_a \lambda^a \qquad \bar{B} = \frac{1}{\sqrt{2}} \bar{B}_a \lambda^a \;. \tag{2.12}$$

Since we are only interested in transitions involving at most two baryons, we restrict ourselves to terms in $\mathcal{L}_{HM}$, which are bilinear in the baryon field B. The Lagrangian $\mathcal{L}_{HM}$ can now be brought to a convenient form by extracting the baryon fields explicitly

$$\mathcal{L}_{HM} = \bar{B}^a D^{ab} B^b \;. \tag{2.13}$$

$D^{ab}(U;v,a,s,p)$ is a differential operator containing only the meson field U and the external fields; an outline is given in the next chapter.

Adding external Grassmann sources for the baryon fields to $\mathcal{L}$

$$\mathcal{L} = \mathcal{L}_{Mes} + \mathcal{L}_{HM} + \bar{\eta}^a B^a + \bar{B}^a \eta^a \ ,$$

we consider the vacuum transition amplitude [15]

$$\begin{aligned} < 0_{out}|0_{in} >_{v,a,s,p;\eta,\bar{\eta}} &= e^{i\mathcal{Z}(v,a,s,p;\eta,\bar{\eta})} \\ &= \int \mathcal{D}U\ \mathcal{D}B\ \mathcal{D}\bar{B}\ e^{i\int d^4x\ \mathcal{L}(B,\bar{B},U;v,a,s,p;\eta,\bar{\eta})} \\ &= \int \mathcal{D}U e^{i\int d^4x\ \mathcal{L}_{Mes} - i\int d^4x \int d^4y\ \bar{\eta}^a(x) S_{ab}(x,y)\eta^b(y)} \det(D)\ . \end{aligned} \tag{2.14}$$

Here $S^{ab}(x,y|U;v,a,s,p)$ is the baryon propagator in the presence of the meson fields and external fields; it is the inverse of the differential operator D

$$D^{ac}S^{cb} = \delta^4(x-y)\delta^{ab}\ .$$

The generating functional $\mathcal{Z}$ coincides with the generating functional Z introduced earlier, when $\eta = \bar{\eta} = 0$ and $\det D = 1$. By definition, the second derivative of $\mathcal{Z}$ coincides with the the baryon propagator $\mathcal{S}$ in the presence of external fields

$$\mathcal{S}(x,y|v,a,s,p) = \frac{\delta}{i\delta\eta(x)}\frac{\delta}{i\delta\bar{\eta}(y)}\mathcal{Z}\,|_{\eta=\bar{\eta}=0}\ . \tag{2.15}$$

Note the difference between $\mathcal{S}$ and S, defined above, where S also depends on the meson field. The Fourier transform of $\mathcal{S}$,

$$\tilde{\mathcal{S}}(p,p'|v,a,s,p) = \int d^4x \int d^4y\ e^{ipx - ip'y}\mathcal{S}(x,y|v,a,s,p) \tag{2.16}$$

has poles at $p^2 = M_P^2, p'^2 = M_Q^2$. M_P and M_Q denotes the mass of the incoming and outgoing baryon respectively. The residue is proportional to the baryon-baryon transition amplitude $\mathcal{F}(p,p'|v,a,s,p)$ in the presence of external fields [15]

$$\mathcal{F}(p,p'|v,a,s,p) =< p_{out}|p'_{in} >_{v,a,s,p}\ . \tag{2.17}$$

In the introduction and the first chapter we have introduced the baryon matrix elements of the vector and axialvector currents. The relation between the amplitude $\mathcal{F}$ and these matrix elements is given by

$$\begin{aligned} < p|\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)|p' > &= \frac{\delta}{i\delta v_\mu^a(x)}\mathcal{F}\,|_{v=0} \\ < p|\bar{q}(x)\gamma^5\gamma^\mu\frac{\lambda^a}{2}q(x)|p' > &= \frac{\delta}{i\delta a_\mu^a(x)}\mathcal{F}\,|_{a=0}\ . \end{aligned} \tag{2.18}$$

The knowledge of the low energy behaviour of $\mathcal{Z}$ or $\mathcal{S}$ can thus be used to obtain an expression for the amplitude $\mathcal{F}$ and the form factors for small momenta. The path integral representation of $\mathcal{Z}$ in terms of an effective Lagrangian allows one to determine the low energy structure of $\mathcal{Z}$.

When one is dealing exclusively with mesons, the low energy expansion of $\mathcal{Z}$ reduces to the loop expansion of the effective meson theory. In this case n–loop contributions are suppressed by a factor of $O(q^{2n})$. For a proof of this statement see Weinberg [32]. In the presence of baryons the situation is somewhat different. The mass of the baryons is of $O(1)$ and goes to a constant M_0 in the chiral limit, as we will see later. Contributions from chiral loops therefore contain a piece, which is not suppressed. However loops containing only fermions as well as higher order loops will not contribute to the leading singularities. For a more detailed discussion we refer to the thesis [21]. Since we are interested in the leading singularities, we restrict ourselves to one loop diagrams, omitting closed baryon loops. This amounts to the approximation $\det D = 1$ in the path integral representation of $\mathcal{Z}$ (no integration over B, $\bar{B}$).

2.3 Baryon propagator

In the last section we have given the path integral representation of $\mathcal{Z}$. This also induces a path integral representation for the baryon propagator $\mathcal{S}$. In this section we give another representation of $\mathcal{S}$ in terms of quantized fields, which will be used in our calculations. We decompose the effective Lagrangian into three pieces

$$\mathcal{L} = \mathcal{L}^0_{Mes} + \mathcal{L}^0_{Hyp} + \mathcal{L}_{int} \; ,$$

where $\mathcal{L}^0_{Mes}$ is the Lagrangian of a free scalar field, $\mathcal{L}^0_{Hyp}$ is the Lagrangian of a free Dirac spinor field and $\mathcal{L}_{int}$ describes the interaction between the scalar field (mesons) and the spinor field (baryons). $\mathcal{L}_{int}$ is a function of B, $\bar{B}$, U and the external fields; B, $\bar{B}$ and U have been introduced in the last two sections. We quantize the theory in the standard manner: the baryon field B and the meson field U are quantized according to the standard rules for spin 1/2 and spin 0 particles respectively acting on the Hilbert space of states, which is generated by a Lorentz invariant vacuum state $|0>$. A representation of $\mathcal{S}$ is now given by the Gell-Mann–Low formula [5]

$$\begin{aligned} \mathcal{S} &= <0|TB^a(x)\bar{B}^b(y)|0> \\ &= <0_{in}|TB^a_{in}(x)\bar{B}^b_{in}(y)e^{i\int d^4x\, \mathcal{L}_{int}(B_{in},\bar{B}_{in},U_{in};v,a,s,p)}|0_{in}>_{connected} \; . \end{aligned} \tag{2.19}$$

This representation of $\mathcal{S}$ will be used later to obtain a low energy representation of the baryon-baryon transition amplitude $\mathcal{F}$ and the form factors. In order to be able to use this representation we first have to construct the effective baryon-meson Lagrangian $\mathcal{L}_{HM}$. This will be done in the next chapter.

Chapter 3

Construction of the Effective Chiral Lagrangian

3.1 Field content

In the last chapter we have shown how an effective Lagrangian can be used to derive the low energy behaviour of the baryon-baryon transition amplitude. There we introduced a meson field U, a baryon field B and a set of external fields. Now we examine these fields in more detail. In the effective meson theory the meson field U is described by a unitary 3×3 matrix in flavour space

$$U^\dagger U = \mathbf{1} \qquad \det U = 1\,, \tag{3.1}$$

transforming linearly under chiral transformations

$$U' = V_R U V_L^\dagger \qquad V_R, V_L \in SU(3)\,. \tag{3.2}$$

The covariant derivative of U in the linear representation, ∇_μ, is given by [16,17]

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - i\tilde{F}^R_\mu U + iU\tilde{F}^L_\mu \\ \nabla_\mu U^\dagger &= \partial_\mu U^\dagger + iU^\dagger \tilde{F}^R_\mu - i\tilde{F}^L_\mu U^\dagger\,, \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} \tilde{F}^R_\mu &= v_\mu + a_\mu \\ \tilde{F}^L_\mu &= v_\mu - a_\mu\,. \end{aligned} \tag{3.4}$$

The baryon field transforms however nonlinearly under chiral transformations (see equation 2.10). When constructing the effective Lagrangian, it turns out to be more convenient to work with fields, that have the same transformation law; we will use only nonlinearly transforming fields. In order to achieve this, we have to introduce meson and external fields transforming like B. The mesons are collectsd in the field Δ_μ [16,17]

$$\begin{aligned} \Delta_\mu &= \tfrac{1}{2}u^\dagger(\nabla_\mu U)u^\dagger \\ &= -\tfrac{1}{2}u(\nabla_\mu U^\dagger)u\,, \end{aligned} \tag{3.5}$$

where $U = u^2$. One can indeed show that

$$\Delta'_\mu = R\Delta_\mu R^\dagger \,. \tag{3.6}$$

We first define a set of fields related to the external fields v, a.

$$\begin{aligned} \tilde{F}^R_{\mu\nu} &= \partial_\mu \tilde{F}^R_\nu - \partial_\nu \tilde{F}^R_\mu - i[\tilde{F}^R_\mu, \tilde{F}^R_\nu] \\ \tilde{F}^L_{\mu\nu} &= \partial_\mu \tilde{F}^L_\nu - \partial_\nu \tilde{F}^L_\mu - i[\tilde{F}^L_\mu, \tilde{F}^L_\nu] \,. \end{aligned} \tag{3.7}$$

$\tilde{F}^{R,L}_{\mu\nu}$ denotes the field strength tensor corresponding to the fields $\tilde{F}^{R,L}_\mu$ introduced above. We further define

$$\begin{aligned} F^R_{\mu\nu} &= u^\dagger \tilde{F}^R_{\mu\nu} u \\ F^L_{\mu\nu} &= u \tilde{F}^L_{\mu\nu} u^\dagger \,, \end{aligned} \tag{3.8}$$

where $F^{R,L}_{\mu\nu}$ denotes now the nonlinearly transforming field strength tensor; the parity even and odd combinations are

$$\begin{aligned} F^+_{\mu\nu} &= F^R_{\mu\nu} + F^L_{\mu\nu} \\ F^-_{\mu\nu} &= F^R_{\mu\nu} - F^L_{\mu\nu} \,. \end{aligned} \tag{3.9}$$

The scalar field s is replaced by

$$\sigma = \frac{1}{2}(u\chi^\dagger u + u^\dagger \chi u^\dagger) \,, \tag{3.10}$$

where $\chi = 2B_0(s + ip)$ with B_0 being a constant occurring in the meson field theory, whereas

$$\varrho = \frac{1}{2}(u\chi^\dagger u - u^\dagger \chi u^\dagger) \tag{3.11}$$

replaces the pseudoscalar field p. All the fields B, $\Delta_\mu, F^+_{\mu\nu}, F^-_{\mu\nu}, \sigma, \varrho$ have the same transformation law under chiral transformations

$$X' = RXR^\dagger \,. \tag{3.12}$$

In this representation the covariant derivative is given by

$$[D_\mu, X] = \partial_\mu X + [\Gamma_\mu, X] \,, \tag{3.13}$$

with

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}(F^R_\mu + F^L_\mu) \tag{3.14}$$

as the connection; the fields $F^{R,L}_\mu$ are defined similar to $F^{R,L}_{\mu\nu}$

$$\begin{aligned} F^R_\mu &= u^\dagger \tilde{F}^R_\mu u \\ F^L_\mu &= u \tilde{F}^L_\mu u^\dagger \,. \end{aligned} \tag{3.15}$$

In equation (3.13) we have introduced an operator form of the covariant derivative. A few remarks about the operator D_μ are useful:

- D_μ should be viewed as a differential operator as well as a matrix in flavour space
- D_μ acts on everything on its right
- D_μ can formally be written as
$$D_\mu = \partial_\mu + \Gamma_\mu$$
- D_μ can be treated like any other field
- the use of an operator D_μ is convenient in deriving relations among covariant derivatives of the fields.

The covariant derivative of a field in the nonlinear representation can be expressed in terms of the covariant derivative in the linear representation. As an example we state this relation for $[D_\mu, \Delta_\nu]$. After some algebra one obtains

$$\begin{aligned} [D_\mu, \Delta_\nu] &= (\partial_\mu \Delta_\nu) + \Gamma_\mu \Delta_\nu - \Delta_\nu \Gamma_\mu \\ &= \tfrac{1}{4} u^\dagger (\nabla_\mu \nabla_\nu U) u^\dagger - \tfrac{1}{4} u (\nabla_\mu \nabla_\nu U^\dagger) u \, . \end{aligned} \tag{3.16}$$

Similar results can be obtained for the covariant derivative of the other fields. In addition we find two identities between covariant derivatives and fields in the nonlinear representation, which will prove very useful for the construction of the effective Lagrangian

$$\begin{aligned} [D_\mu, \Delta_\nu] - [D_\nu, \Delta_\mu] &= -\tfrac{i}{2} F^-_{\mu\nu} \\ [D_\mu, D_\nu] &= -[\Delta_\mu, \Delta_\nu] - \tfrac{i}{2} F^+_{\mu\nu} \, . \end{aligned} \tag{3.17}$$

We now examine the properties of the fields under Lorentz- and parity transformations:

$$\begin{aligned}
\bar{B}' B' &= \bar{B} B && (scalar) \\
\bar{B}' \gamma_5 B' &= \det\Lambda \; \bar{B} \gamma_5 B && (pseudoscalar) \\
\bar{B}' \gamma_\mu B' &= \Lambda_\mu^{\ \nu} \; \bar{B} \gamma_\nu B && (vector) \\
\bar{B}' \gamma_5 \gamma_\mu B' &= \det\Lambda \; \Lambda_\mu^{\ \nu} \; \bar{B} \gamma_5 \gamma_\nu B && (pseudovector) \\
\bar{B}' \sigma_{\mu\nu} B' &= \Lambda_\mu^{\ \alpha} \Lambda_\nu^{\ \beta} \; \bar{B} \sigma_{\alpha\beta} B && (tensor) \\
\sigma' &= \sigma && (scalar) \\
\varrho' &= \det\Lambda \; \varrho && (pseudoscalar) \\
D'_\mu &= \Lambda_\mu^{\ \nu} \; D_\nu && (vector) \\
\Delta'_\mu &= \det\Lambda \; \Lambda_\mu^{\ \nu} \; \Delta_\nu && (pseudovector) \\
F^{+\prime}_{\mu\nu} &= \Lambda_\mu^{\ \alpha} \Lambda_\nu^{\ \beta} \; F^+_{\alpha\beta} && (tensor) \\
F^{-\prime}_{\mu\nu} &= \det\Lambda \; \Lambda_\mu^{\ \alpha} \Lambda_\nu^{\ \beta} \; F^-_{\alpha\beta} && (pseudotensor)
\end{aligned} \tag{3.18}$$

The relations involving B are well known. In order to verify the properties for the other fields, we look at the meson field in the linear representation U, parametrized by

$$U = e^{i\Phi} \, .$$

Since the mesons are pseudoscalar particles, they change sign under parity transformations $\Phi' = -\Phi$. This implies

$$U' = U^\dagger \qquad u' = u^\dagger \, .$$

The properties of the fields in the nonlinear representation, (3.18), can now be derived using the definition of the covariant derivative ∇_μ in the linear representation (3.3) and the properties of the external fields.

Next we consider the charge conjugation properties of the fields. The baryon field B is a Dirac spinor; the charge conjugate field B^c is given by

$$B^c = C\bar{B}^T \, , \tag{3.19}$$

where C is the usual charge conjugation matrix. For any element of the Clifford algebra Γ we have (see appendix A)

$$C\Gamma C^{-1} = (-1)^{c_\Gamma}\Gamma^T \, . \tag{3.20}$$

c_Γ can be interpreted as the charge conjugation of the element Γ. For bilinear forms in the Clifford algebra this implies

$$(\bar{B}\Gamma B)^c = \bar{B}^c\Gamma B^c = (-1)^{c_\Gamma}(\bar{B}\Gamma B) \, ,$$

where we have used

$$B^T\Gamma^T\bar{B}^T = -(\bar{B}\Gamma B) \, .$$

Demanding charge conjugation invariance for the QCD Lagrangian given in (2.2), we find the following properties of the external fields

$$\begin{aligned} s^c &= s^T & \qquad p^c &= p^T \\ v_\mu^c &= -v_\mu^T & \qquad a_\mu^c &= a_\mu^T \, . \end{aligned}$$

For instance

$$(\bar{q}\gamma^\mu v_\mu q)^c = (\bar{q}\gamma^\mu v_\mu q)$$

implies $v_\mu^c = -v_\mu^T$. The meson field Φ behaves like the pseudoscalar external field p:

$$\Phi^c = \Phi^T \, ;$$

this yields $U^c = U^T, u^c = u^T$. Thus we obtain the following properties for the fields in the nonlinear representation

$$\begin{aligned} \Delta_\mu^c &= \Delta_\mu^T \\ D_\mu^c &= -D_\mu^T \\ F_{\mu\nu}^{+\,c} &= -F_{\mu\nu}^{+\,T} \\ F_{\mu\nu}^{-\,c} &= F_{\mu\nu}^{-\,T} \\ \sigma^c &= \sigma^T \\ \varrho^c &= \varrho^T \, . \end{aligned} \tag{3.21}$$

Finally we examine the hermiticity properties of the fields. We have

$$(\bar{B}\Gamma B)^\dagger = B^\dagger\Gamma^\dagger\gamma^0 B = \bar{B}(\gamma^0\Gamma^\dagger\gamma^0)B \,. \tag{3.22}$$

For any element of the Clifford algebra the following relation holds (see appendix A)

$$\gamma^0\Gamma^\dagger\gamma^0 = (-1)^{h_\Gamma}\Gamma \,, \tag{3.23}$$

implying

$$(\bar{B}\Gamma B)^\dagger = (-1)^{h_\Gamma}(\bar{B}\Gamma B) \,. \tag{3.24}$$

For the fields in the nonlinear representation one thus obtains

$$\begin{aligned} \Delta_\mu^\dagger &= -\Delta_\mu \\ D_\mu^\dagger &= -D_\mu \\ F_{\mu\nu}^{+\,\dagger} &= F_{\mu\nu}^{+} \\ F_{\mu\nu}^{-\,\dagger} &= F_{\mu\nu}^{-} \\ \sigma^\dagger &= \sigma \\ \varrho^\dagger &= -\varrho \,. \end{aligned} \tag{3.25}$$

3.2 Chiral power counting

In this section we introduce the concept of chiral power counting. It is very useful in classifying the terms in the effective Lagrangian and therefore also in obtaining a coherent low energy representation of the generating functional introduced in chapter 2. First we look at the mesons. In the chiral limit the mass of the mesons is zero. If the spatial momentum of the mesons is small, then this will also be the case for the four-momentum; in this case we have a genuine small four-momentum q, which can be used for power counting.

If we count the meson field U as a quantity of $O(1)$, then the derivative $\partial_\mu U$ is of $O(q)$. Since the external fields v, a occur linearly in the covariant derivative $\nabla_\mu U$, it is convenient to count them as $O(q)$. As we pointed out in the last chapter the low energy expansion of the generating functional is a double expansion in the momentum and in the quark mass matrix $\mathcal{M}$, with fixed ratio $\mathcal{M}/q^2$. Thus $\mathcal{M}$ counts as $O(q^2)$. Since the quark mass matrix is contained in the external field s, we have to count the scalar field s as a quantity of $O(q^2)$ as well. The field p occurs in the combination $s + ip$ in the transformation law with respect to chiral transformations; it is conveniently booked as $O(q^2)$. To summarize we count the fields as follows :

$$U, u = O(1) \qquad a_\mu, v_\mu = O(q) \qquad s, p = O(q^2) \,.$$

For the external fields in the nonlinear representation this then yields

$$\begin{gathered} \Delta_\mu = O(q) \qquad F_{\mu\nu}^{+} = O(q^2) \qquad F_{\mu\nu}^{-} = O(q^2); \\ \sigma = O(q^2) \qquad \varrho = O(q^2) \,. \end{gathered} \tag{3.26}$$

D_μ is counted as $O(q)$, when it acts on any of these fields, because the connection and the derivative ∂_μ both are of this order. However it is not *a priori* clear how to count the baryon field B and the covariant derivative thereof. Since the mass of the baryon does not vanish in the chiral limit it is not straightforward to extend these power counting rules to the field B. It does not make sense to treat the four-momentum of the baryons as a small quantity, because the mass of these particles is not small compared to the scale of the theory.

In order to examine this problem we look at the nonrelativistic limit of the baryons. Since the connection Γ_μ is anyhow of $O(q)$, we only have to look at the case $D_\mu = \partial_\mu$. Moreover we can switch off all the other fields, since they are at least of $O(q)$; that is we can deal with free baryons. The baryon field B is a four component Dirac spinor. We write [20]

$$B = \begin{pmatrix} \phi \\ \chi \end{pmatrix} ,$$

thereby introducing large and small components in the nonrelativistic limit. The Dirac equation then reads

$$\begin{aligned} i\frac{\partial\phi}{\partial t} &= (\sigma_k \hat{p}_k)\, \chi + M_0\, \phi \\ i\frac{\partial\chi}{\partial t} &= (\sigma_k \hat{p}_k)\, \phi - M_0\, \chi \,, \end{aligned} \tag{3.27}$$

where σ_k denotes the Pauli matrices and $\hat{p}_k$ is the three momentum operator. The mass M_0 is the driving term in these equations. By introducing slowly varying functions of time

$$\begin{aligned} \phi &= e^{-iM_0 t}\tilde{\phi} \\ \chi &= e^{-iM_0 t}\tilde{\chi} \,, \end{aligned} \tag{3.28}$$

one can solve these equations approximately, yielding

$$\begin{aligned} \hat{\chi} &= \frac{\sigma_k \hat{p}_k}{2M_0}\,\hat{\phi} \\ i\frac{\partial\hat{\phi}}{\partial t} &= \frac{(\sigma_k \hat{p}_k)^2}{2M_0}\,\hat{\phi} \,. \end{aligned} \tag{3.29}$$

We now conclude: for small three-momentum of the baryons $\hat{\chi}$ is suppressed relative to $\hat{\phi}$ and this is also true for χ relative to ϕ. The action of $(iD_0 - M_0) = (i\frac{\partial}{\partial t} - M_0)$ on ϕ and χ is given by

$$\begin{aligned} (iD_0 - M_0)\phi &= \sigma_k \hat{p}_k\, \chi \\ (iD_0 - M_0)\chi &= \sigma_k \hat{p}_k\, \phi - 2M_0\, \chi \,. \end{aligned} \tag{3.30}$$

The operator $(iD_0 - M_0)$ therefore is booked as $O(q)$, whereas D_0 itself has to be counted as $O(1)$.

The elements of the Clifford algebra Γ in general mix the small and large components of the Dirac spinor B. It is therefore convenient to introduce the following counting rules for the Γ:

$$\mathbf{1}, \gamma_0, \gamma_5\gamma_k, \sigma_{kl} = O(1) \qquad \gamma_5, \gamma_k, \gamma_5\gamma_0, \sigma_{k0} = O(q) \,. \tag{3.31}$$

For instance $\bar{B}\gamma_5 B$ contains at least one small component in the nonrelativistic limit.

According to the discussion above we are thus lead to count the relativistic fields and the elements of the Clifford algebra as follows:

$$B, \bar{B} = O(1) \qquad [D_\mu, B] = O(1) \,, \qquad i\gamma^\mu[D_\mu, B] - M_0 B = O(q) \,, \tag{3.32}$$

$$\mathbf{1}, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu} = O(1) \qquad \gamma_5 = O(q) \,. \tag{3.33}$$

The order associated with these operators is the minimal one; some of them, like $[D_\mu, B]$, contain in addition a piece of higher order.

3.3 Construction of the effective baryon-meson Lagrangian

In this section we describe a scheme to construct the baryon-meson Lagrangian $\mathcal{L}_{HM}$. We require $\mathcal{L}_{HM}$ to be a real, flavour neutral, scalar and invariant with respect to chiral transformations,proper Lorentz transformations as well as C,P,T. For the construction we use fields transforming nonlinearly; all of them are 3×3 matrices in flavour space. $\mathcal{L}_{HM}$ is a polynomial in these fields and derivatives thereof. According to a theorem of Weyl all invariant polynomials can be written as traces over the flavour group. In addition one has to form bilinear forms in the Clifford algebra to obtain a scalar in Dirac space. Thus a general term, containing only one trace has the form

$$\mathrm{tr}(A_1 \bar{B}\Gamma A_2 B A_3) \,. \tag{3.34}$$

A_1, A_2, A_3 denotes any combination of the fields $\varrho, \sigma, \Delta_\mu, F^+_{\mu\nu}, F^-_{\mu\nu}$ and the covariant derivative D_μ; Γ is an element of the Clifford algebra. For the moment we restrict ourselves to this case; the generalization to more than one trace will be done later. Using the cyclic property of the trace, one can always bring the field $\bar{B}$ to the very left, leaving

$$\mathrm{tr}(\bar{B}\Gamma A_1 B A_2) \tag{3.35}$$

as a general term for $\mathcal{L}_{HM}$. Next to ensure invariance with respect to proper Lorentz transformations and parity, the tensor indices have to be properly contracted with $g_{\mu\nu}$ or $\epsilon_{\mu\nu\alpha\beta}$.

We still have to check, if a term constructed with this recipe is real and charge conjugation invariant. For this purpose we write the general term in a somewhat different way

$$\mathrm{tr}(\bar{B}\Gamma(A_1, (A_2 \cdots, (A_n, B) \cdots)) \,) \,, \tag{3.36}$$

where (A_1, A_2) denotes either the commutator $[A_1, A_2]$ or the anticommutator $\{A_1, A_2\}$ of the fields A_1 and A_2. The $A_1, \ldots, A_n$ can be any field except B, $\bar{B}$ or a combination of (anti-) commutators thereof. This form turns out to be more suitable; it is of course equivalent to equation (3.35). Now we derive some useful formulae.

We first investigate the charge conjugation invariance.

$$\begin{aligned} \mathrm{tr}(\bar{B}\Gamma(A_1,(A_2\cdots,(A_n,B)\cdots))\,)^c &= \mathrm{tr}(\bar{B}^c\Gamma(A_1^c,(A_2^c\cdots,(A_n^c,B^c)\cdots))\,) \\ &= (-1)^{c_1+\cdots+c_n+c_\Gamma+1} \\ &\quad \cdot\mathrm{tr}((A_1,(A_2\cdots,(A_n,B)\cdots))^T\Gamma^T\bar{B}^T\,) \\ &= (-1)^{c_1+\cdots+c_n+c_\Gamma} \\ &\quad \cdot\mathrm{tr}(\bar{B}\Gamma(A_n,\cdots,(A_2,(A_1,B))\cdots)\,)\,, \end{aligned} \tag{3.37}$$

where we have used (see appendix A)

$$C\Gamma C^{-1} = (-1)^{c_\Gamma}\Gamma^T\,, \tag{3.38}$$

$$A_k^c = (-1)^{c_k}A_k^T\,. \tag{3.39}$$

The last formula is valid, because A_k is either a field or a combination of (anti-) commutators thereof; whereas for the product of two fields A_1, A_2 one obtains

$$(A_1\cdot A_2)^c = (A_1^c\cdot A_2^c) = \pm(A_1^T\cdot A_2^T) \neq \pm(A_1\cdot A_2)^T\,.$$

This is the reason why we will use equation (3.36) for a general term of $\mathcal{L}_{HM}$.

Next we look at the hermiticity property of such a term.

$$\begin{aligned} \mathrm{tr}(\bar{B}\Gamma(A_1,(A_2\cdots,(A_n,B)\cdots))\,)^* &= \mathrm{tr}(\,(\cdots(\bar{B},A_n^\dagger)\cdots A_1^\dagger)\gamma_0\Gamma^\dagger\gamma_0 B\,) \\ &= (-1)^{h_1+\cdots+h_n+h_\Gamma} \\ &\quad \cdot\mathrm{tr}(\bar{B}\Gamma(A_n,\cdots,(A_2,(A_1,B))\cdots)\,)\,, \end{aligned} \tag{3.40}$$

where we have used (see appendix A)

$$\gamma_0\Gamma^\dagger\gamma_0 = (-1)^{h_\Gamma}\Gamma\,, \tag{3.41}$$

$$A_k^\dagger = (-1)^{h_k}A_k\,. \tag{3.42}$$

The trace occurring in the last line of the equations (3.37,3.40) can be brought to a form, in which the A_k are ordered differently, using the cyclic property of the trace and the following important identities:

$$\begin{aligned} [A,[C,B]] &= [C,[A,B]] + [[A,C],B] \\ [A,\{C,B\}] &= \{C,[A,B]\} + \{[A,C],B\} \\ \{A,\{C,B\}\} &= \{C,\{A,B\}\} + [[A,C],B]\,; \end{aligned} \tag{3.43}$$

the first one is the well known Jacobi identity. We then obtain

$$\begin{aligned}\mathrm{tr}(\bar{B}\Gamma(A_n,\cdots,(A_2,(A_1,B))\cdots)\,) &= \mathrm{tr}(\bar{B}\Gamma(A_1,(A_2\cdots,(A_n,B)\cdots))\,)+\\ &\quad \mathrm{tr}(\bar{B}\Gamma(\tilde{A}_1,(\tilde{A}_2\cdots,(\tilde{A}_m,B)\cdots))\,)\,,\end{aligned} \tag{3.44}$$

where $\tilde{A}_1,\ldots,\tilde{A}_m$ are (anti-) commutators of the A_k and where $m < n$. As an example we state

$$\mathrm{tr}(\bar{B}\Gamma[A_2,\{A_1,B\}]\,) = \mathrm{tr}(\bar{B}\Gamma[A_1,\{A_2,B\}]\,) + \mathrm{tr}(\bar{B}\Gamma\{[A_2,A_1],B\}\,)\,.$$

In this case $\tilde{A}_1 = [A_2, A_1]$. To make sure that a general term X of $\mathcal{L}_{HM}$ is real and charge conjugation even, we take the combinations $\frac{1}{2}(X + X^*)$ and $\frac{1}{2}(X + X^c)$. From the equations (3.37,3.40,3.43) we see, that X will *not* drop out in these combinations only if

$$(-1)^{c_1+\cdots+c_n+c_\Gamma} = 1\,, \tag{3.45}$$

$$(-1)^{h_1+\cdots+h_n+h_\Gamma} = 1\,. \tag{3.46}$$

If these conditions are fulfilled, $\frac{1}{2}(X + X^c)$ is an allowed term in $\mathcal{L}_{HM}$.

We now discuss the extension to more than one trace in flavour space. The recipe described so far can still be used, however with a few changes. A general term has now either of the following forms:

$$\mathrm{tr}(\bar{B}\Gamma(A_1,(A_2\cdots,(A_n,B)\cdots))\,)\cdot\mathrm{tr}(C_1) \tag{3.47}$$

$$\mathrm{tr}(\bar{B}A_1)\Gamma\mathrm{tr}(A_2B)\cdot\mathrm{tr}(C_1)\,. \tag{3.48}$$

If the fields B and $\bar{B}$ are not embraced by one trace, the equations (3.37) and (3.40) have to be replaced by

$$[\mathrm{tr}(\bar{B}A_1)\Gamma\mathrm{tr}(A_2B)]^c = (-1)^{c_1+c_2+c_\Gamma}\cdot\mathrm{tr}(\bar{B}A_2)\Gamma\mathrm{tr}(A_1B) \tag{3.49}$$

$$[\mathrm{tr}(\bar{B}A_1)\Gamma\mathrm{tr}(A_2B)]^* = (-1)^{h_1+h_2+h_\Gamma}\cdot\mathrm{tr}(\bar{B}A_2)\Gamma\mathrm{tr}(A_1B)\,. \tag{3.50}$$

A term of the form $\frac{1}{2}(X + X^c)$ can now always be accepted even if the condition (3.45) is not fulfilled, since X will never drop out. The formal use of the covariant derivative D_μ must now be handled with care. As an example we give

$$\mathrm{tr}([D_\mu,\bar{B}]\Gamma B)\cdot\mathrm{tr}(A_1) = -\mathrm{tr}(\bar{B}\Gamma[D_\mu,B])\cdot\mathrm{tr}(A_1) - \mathrm{tr}(\bar{B}\Gamma B)\cdot\mathrm{tr}([D_\mu,A_1])\,;$$

note the presence of the last term. $\mathrm{tr}(C_1)$, the last trace in (3.47,3.48), can be any term of the effective meson Lagrangian $\mathcal{L}_{Mes}$. The general form of $\mathcal{L}_{Mes}$ up to $O(q^4)$ has been worked out by Leutwyler and Gasser [16,17]. We write $\mathcal{L}_{Mes}$ as a sum

$$\mathcal{L}_{Mes} = \mathcal{L}^2_{Mes} + \mathcal{L}^4_{Mes} + \cdots\,, \tag{3.51}$$

where the upper index denotes the chiral order. At lowest order we find

$$\mathcal{L}^2_{Mes} = -\frac{1}{2}F_0^2 \cdot \mathrm{tr}(\{\Delta_\mu, \Delta^\mu\}\ ) + \frac{1}{2}F_0^2 \cdot \mathrm{tr}(\sigma)\ , \tag{3.52}$$

where F_0 is the meson decay constant.

Using the recipe described above, we can construct all the allowed terms of $\mathcal{L}_{HM}$. To each of them we can assign a chiral order, using the rules of chiral power counting discussed in the last section. We write $\mathcal{L}_{HM}$ as a sum

$$\mathcal{L}_{HM} = \mathcal{L}^1_{HM} + \mathcal{L}^2_{HM} + \mathcal{L}^3_{HM} + \cdots\ ; \tag{3.53}$$

again the upper index denotes the chiral order. We are thus able to list all terms in the effective Lagrangian up to a given chiral order. However not all the terms we get this way will be independent. In order to find a minimal set of independent terms we use the identities of equation (3.43) and relations among elements of the Clifford algebra, given in appendix B.

Finally we derive the connection between the differential operator D, defined in chapter 2, and the representation of terms involving traces over the flavour group, given above. Remembering that

$$B = \frac{1}{\sqrt{2}}B_a\lambda^a \qquad \bar{B} = \frac{1}{\sqrt{2}}\bar{B}_a\lambda^a$$

we obtain

$$\begin{aligned}
&\mathrm{tr}(\bar{B}\Gamma(A_1,(A_2\cdots,(A_n,B)\cdots))\ ) \\
&= \frac{1}{2}\mathrm{tr}(\lambda^a(A_1,(A_2\cdots,(A_n,\lambda^b)\cdots))\ )\cdot\bar{B}_a\Gamma B_b \\
&= \bar{B}_a D^{ab} B_b\ .
\end{aligned}$$

The differential D is therefore given by

$$D^{ab} = \frac{1}{2}\,\Gamma\cdot\mathrm{tr}(\lambda^a(A_1,(A_2\cdots,(A_n,\lambda^b)\cdots))\ )$$

and analogously for terms involving more than one trace. D is thus also given by the above recipe.

3.4 Effective Lagrangian to $O(q)$

In this section we give an explicit construction for the lowest order term $\mathcal{L}^1_{HM}$. We start with contributions containing only the baryon field B and covariant derivatives thereof. Following the recipe of the last section we find two independent terms, leading to

$$\mathcal{L}^1_{HM} = a\cdot\mathrm{tr}(\bar{B}i\gamma^\mu[D_\mu,B]\ ) + b\cdot\mathrm{tr}(\bar{B}B\ )\ , \tag{3.54}$$

where a and b are free constants. The first contribution contains the kinetic term. Without loss of generality we can choose $a = 1$, since the field B can always be rescaled. The second contribution is a mass term. If we choose $b = -M_0$ and switch off all other fields, the Lagrangian $\mathcal{L}^1_{HM}$ reduces to the free Dirac Lagrangian. The parameter M_0 is the mass of the baryon in the chiral limit (the quark mass matrix $\mathcal{M}$ is switched off). Using the rules for chiral power counting, given in section 3.2, the resulting Lagrangian

$$\mathcal{L}_{HM} = \mathrm{tr}(\bar{B} i\gamma^\mu [D_\mu, B]\,) - M_0 \cdot \mathrm{tr}(\bar{B}B\,) \tag{3.55}$$

has to be counted as $O(q)$. Other terms of the same order can be constructed using the field Δ_μ. One finds

$$\mathcal{L}_{HM} = D \cdot \mathrm{tr}(\bar{B} i\gamma^5\gamma^\mu \{\Delta_\mu, B\}\,) + F \cdot \mathrm{tr}(\bar{B} i\gamma^5\gamma^\mu [\Delta_\mu, B]\,)\ , \tag{3.56}$$

where D and F are free parameters. Obviously combinations involving B, Δ_μ as well as other fields will give rise to higher order terms. However there might exist a term involving Δ_μ and $[D_\mu, B]$ or $[D_\nu, [D_\mu, B]]$ at that order. In the next section we examine such terms. We will see, that the complete effective baryon-meson Lagrangian to first order is given by

$$\begin{aligned} \mathcal{L}^1_{HM} \;=\; & \mathrm{tr}(\bar{B} i\gamma^\mu [D_\mu, B]\,) - M_0 \cdot \mathrm{tr}(\bar{B}B\,) + \\ & D \cdot \mathrm{tr}(\bar{B} i\gamma^5\gamma^\mu \{\Delta_\mu, B\}\,) + F \cdot \mathrm{tr}(\bar{B} i\gamma^5\gamma^\mu [\Delta_\mu, B]\,)\ . \end{aligned} \tag{3.57}$$

3.5 Equation of motion for baryons

In this section we discuss how the equation of motion of the baryons can be used to reduce the number of independent terms in the effective Lagrangian. In chapter 5 we show in an example that terms proportional to the equation of motion do not contribute to the matrix elements of interest. The equation of motion is obtained from the condition $\delta\mathcal{L}_{HM} = 0$. If external baryon sources are present, the equations of motion associated with the Lagrangian $\mathcal{L}^1_{HM}$, given above, read

$$\begin{aligned} i\gamma^\mu [D_\mu, B] - M_0 B + D i\gamma^5\gamma^\mu \{\Delta_\mu, B\} + F i\gamma^5\gamma^\mu [\Delta_\mu, B] + \eta &= 0 \\ -i[D_\mu, \bar{B}]\gamma^\mu - M_0 \bar{B} + D\{\bar{B}, \Delta_\mu\} i\gamma^5\gamma^\mu + F[\bar{B}, \Delta_\mu] i\gamma^5\gamma^\mu + \bar{\eta} &= 0\ . \end{aligned} \tag{3.58}$$

Before stating some general results we look at an example. Let

$$\begin{aligned} x^\alpha &= \mathrm{tr}(\bar{B} g^{\alpha\beta} (A_1, [D_\beta, B])\,) \\ y^\alpha &= \mathrm{tr}(\bar{B} \sigma^{\alpha\beta} (A_1, [D_\beta, B])\,) \end{aligned} \tag{3.59}$$

and suppose x is charge conjugation even; y will then be charge conjugation odd. Using the relations (B.1) of appendix B we find

$$x^\alpha = -i\mathrm{tr}(\bar{B}\gamma^\alpha (A_1, i\gamma^\beta [D_\beta, B] - M_0 B)\,) - iM_0 \cdot \mathrm{tr}(\bar{B}\gamma^\alpha (A_1, B)\,) + y^\alpha\ . \tag{3.60}$$

The second term no longer contains a covariant derivative. The first one is proportional to $i\gamma^\beta[D_\beta, B] - M_0 B$; since this combination counts as $O(q)$, the order of this term is increased relative to x. Moreover it can be eliminated, if the equation of motion can be used (see equation 3.58). Since x is charge conjugation even, we have $x = \frac{1}{2}(x + x^c)$. The insertion of equation (3.60) in the right hand side of this relation leads to the combination

$$y^\alpha + (y^c)^\alpha = -\mathrm{tr}(\bar{B} i\sigma^{\alpha\beta}([D_\beta, A_1], B)) .$$

The covariant derivative here acts on A_1 and thus counts as $O(q)$; the order of $y + y^c$ is increased relative to x. To summarize we see, that x is equivalent to a term with less derivatives acting on B and terms being of higher order, one of which can be eliminated by the use of the equation of motion. This is a more general feature. One can show the following results using the relations in the Clifford algebra, given in appendix B:

1.

$$\mathrm{tr}(\bar{B}\Gamma^{\alpha\beta}(A_1 \cdots, (A_n, [D_\beta, B]) \cdots)) \simeq 0 ,$$

where $\Gamma^{\alpha\beta} = \sigma^{\alpha\beta}$ or $\Gamma^{\alpha\beta} = \gamma^5\sigma^{\alpha\beta}$

2.

$$\mathrm{tr}(\bar{B}\Gamma(A_1 \cdots, (A_n, [D_\beta, B]) \cdots)) \simeq 0 ,$$

where $\Gamma = \mathbf{1}$ or $\Gamma = \gamma^5$

3.

$$\mathrm{tr}(\bar{B} i\Gamma^\beta(A_1 \cdots, (A_n, [D^\alpha, B]) \cdots)) \simeq \mathrm{tr}(\bar{B} i\Gamma^\alpha(A_1 \cdots, (A_n, [D^\beta, B]) \cdots)) ,$$

where $\Gamma^\beta = \gamma^\beta$ or $\Gamma^\beta = \gamma^5\gamma^\beta$

4.

$$\epsilon^{\alpha\beta\mu\nu}\mathrm{tr}(\bar{B}\Gamma_\alpha(A_1 \cdots, (A_n, [D_\beta, B]) \cdots)) \simeq 0$$

5.

$$\epsilon^{\alpha\beta\mu\nu}\mathrm{tr}(\bar{B}\Gamma_{\lambda\alpha}(A_1 \cdots, (A_n, [D_\beta, B]) \cdots)) \simeq 0$$

6.

$$\mathrm{tr}(\bar{B}\Gamma^{\lambda\alpha}(A_1 \cdots, (A_n, [D^\beta, B]) \cdots)) \simeq \mathrm{tr}(\bar{B}\Gamma^{\lambda\beta}(A_1 \cdots, (A_n, [D^\alpha, B]) \cdots)) .$$

Here ' $\simeq$ ' stands for "equivalent to terms with less derivatives acting on B and higher order terms ", as in the example above. Some of the fields A_k could be covariant derivatives. The above formulae can then be used to decrease the number of derivatives acting on B step by step. In particular this can be done with the terms we ignored in the last section. They all reduce to the set of terms of $\mathcal{L}^1_{HM}$, given in equation (3.57), plus higher order terms. $\mathcal{L}^1_{HM}$ is therefore the most general effective baryon-meson Lagrangian of $O(q)$. A complete list of terms contributing to $\mathcal{L}^2_{HM}$ and $\mathcal{L}^3_{HM}$ has been worked out; it is given in appendix B.

Chapter 4

One Loop Feynman Diagrams

4.1 How to use the effective Lagrangian

In the last three chapters we have given the input, necessary to calculate measurable quantities like decay rates and magnetic moments. In the introduction and in chapter 1 we related these quantities to quark current matrix elements; in chapter 2 we discussed the relation between these matrix elements and an effective Lagrangian; in chapter 3 we described a recipe to construct this Lagrangian. Equipped with this information, we now start the calculational part. As was mentioned in chapter 2, the low energy expansion of the generating functional Z in a meson theory coincides with the loop expansion of the effective theory. The loop expansion can also be used for the baryon-meson theory, however there is one difference: in the contribution which is analytic in the quark mass, higher order loops are not suppressed. Tree graphs, which involve low energy constants, also give rise to an analytic contribution. By a suitable renormalization of these constants the analytic part of the loop diagrams can be absorbed. Keeping this in mind, we will evaluate the baryon-meson theory to one loop, neglecting closed fermion loops. The tree contributions will be discussed at the end of this chapter.

The effective Lagrangian consists of a series of terms forming groups of equal chiral powers. In the one loop diagrams the vertices are taken from the baryon-meson Lagrangian $\mathcal{L}_{HM}$ of $O(q)$ and the meson Lagrangian $\mathcal{L}_{Mes}$ of $O(q^2)$. In the free baryon Lagrangian we include a mass term of $O(q^2)$. The standard rules of perturbation theory can be applied; we use the Gell-Mann–Low formula (2.19) to obtain the analytic expressions of the one loop diagrams contributing to the vector current matrix elements.

4.2 Gell–Mann basis versus physical basis

We first look at different bases of $SU(3)$. As was pointed out in chapter 2 the meson field U can be parametrized by a 3×3 hermitean traceless matrix Φ

$$U = e^{i\Phi} .$$

Φ as well as B, the baryon field, can be expanded in a basis of $SU(3)$

$$B = \frac{1}{\sqrt{2}} B_a \lambda^a \qquad \bar{B} = \frac{1}{\sqrt{2}} \bar{B}_a \lambda^a \qquad \Phi = \Phi_a \lambda^a \qquad a = 1, \cdots, 8 .$$

As a basis we either use the Gell–Mann matrices λ^a or the matrices $\tilde{\lambda}^P$, which we call the physical basis. Both occur frequently in the calculations. The transformation from one basis to the other is given by [16,17]

$$\lambda^a = \sum_P N^*_{Pa} \tilde{\lambda}^P = \sum_P N_{Pa} \tilde{\lambda}^{\dagger P} . \tag{4.1}$$

The matrix N is a unitary 8×8 matrix; the non zero elements of N are given in appendix A. In general the matrix N contains the $\Lambda - - \Sigma^0$ mixing. Here we restrict ourselves to the limit $m_u = m_d$; the mixing then disappears. The elements of N are not real; the matrices $\tilde{\lambda}^P$ are therefore not hermitean. Why do we introduce such a basis? Contrary to the Gell–Mann basis, the components of the fields Φ, B, $\bar{B}$ in the physical basis, defined by

$$B = \frac{1}{\sqrt{2}} B_P \tilde{\lambda}^P ; \bar{B} = \frac{1}{\sqrt{2}} \bar{B}_P \tilde{\lambda}^{\dagger P} ; \Phi = \Phi_P \tilde{\lambda}^P \qquad P = 1, \cdots, 8 , \tag{4.2}$$

are directly related to the physical particles. For example B_1 corresponds to the Σ^+. In the quantum theory the operators Φ_P, B_P, when acting on the vacuum, generate the meson states $|\Phi>$ and baryon states $|B>$ respecticely. The signs chosen in N lead to phases in the physical states that are consistent with the De Swart convention [13] (see appendix A). In appendix A one can find the relation between the components of the fields in the two bases.

The free meson- and baryon propagator in the Gell–Mann basis are not diagonal matrices in flavour space. On the other hand the corresponding propagators in the physical basis

$$< 0|T\Phi_P(x)\Phi^\dagger_Q(y)|0 > \;=\; \frac{i}{F_0^2}\, \Delta_{PQ}(x-y; m_P) \tag{4.3}$$

$$< 0|TB_P(x)\bar{B}_Q(y)|0 > \;=\; i\, S_{PQ}(x-y; M_P) , \tag{4.4}$$

are diagonal matrices. Here m_P and M_P denotes the mass of the particle corresponding to the field Φ_P or B_P respectively. In order to show this for the baryon propagator, we examine those terms of $\mathcal{L}^2_{HM}$, that contribute to the mass. These are the following:

$$\mathrm{tr}(\sigma) \cdot \mathrm{tr}(\bar{B}B) \, , \mathrm{tr}(\bar{B}\{\sigma, B\}) \, , \mathrm{tr}(\bar{B}[\sigma, B]) . \tag{4.5}$$

In the physical basis all of them are diagonal; furthermore the term

$$\operatorname{tr}(\bar{B}(i\gamma^\mu\partial_\mu - M_0)B) \tag{4.6}$$

is obviously diagonal. We thus see that the baryon propagator in the physical basis is indeed a diagonal matrix. A similar result can be obtained for the meson propagator using the Lagrangian $\mathcal{L}^2_{Mes}$, given in equation (3.52).

The f- and d-symbols of $SU(3)$ in the Gell–Mann basis are well known. They are defined by

$$f_{abc} = \frac{1}{4i}\operatorname{tr}(\lambda_a[\lambda_b,\lambda_c]) \qquad d_{abc} = \frac{1}{4}\operatorname{tr}(\lambda_a\{\lambda_b,\lambda_c\})\,; \tag{4.7}$$

f_{abc} is real and completely antisymmetric, d_{abc} is real and completely symmetric. We define the corresponding objects in the physical basis by

$$F^R_{PQ} = \frac{1}{4i}\operatorname{tr}(\tilde{\lambda}_P[\tilde{\lambda}_Q,\tilde{\lambda}^\dagger_R]) \qquad D^R_{PQ} = \frac{1}{4}\operatorname{tr}(\tilde{\lambda}_P\{\tilde{\lambda}_Q,\tilde{\lambda}^\dagger_R\})\,; \tag{4.8}$$

The F- and D-symbols obey the following relations:

$$D^R_{PQ} = N_{Pa}N_{Qb}N^*_{Rc}\, d_{abc} \qquad F^R_{PQ} = N_{Pa}N_{Qb}N^*_{Rc}\, f_{abc} \tag{4.9}$$

$$D^R_{PQ} = D^R_{QP} \qquad F^R_{PQ} = -F^R_{QP} \tag{4.10}$$

$$D^{R*}_{PQ} = D^R_{PQ} \qquad {F^R_{PQ}}^* = -F^R_{PQ}\,. \tag{4.11}$$

They are only (anti-) symmetric with respect to the indices P and Q; the last equation shows, that F is imaginary and D is real.

The physical basis has the advantage of leading to diagonal propagators, which are useful in the explicit calculations. On the other hand it is not so convenient to work with the F- and D-symbols in this basis. Thus we use both bases simultaneously, switching from one to the other when convenient.

4.3 Feynman diagrams

In this section we are concerned with the analytic expressions for the Feynman diagrams contributing to the full baryon propagator and the vector current matrix elements at the one loop level. In order to derive them one usually draws all possible diagrams and then assigns an analytic expression to each diagram according to the Feynman rules of that theory. If the theory contains only a few vertices, this will be the most convenient way to proceed. In our case the effective theory contains a lot of different vertices having a complicated structure. We therefore use the Gell–Mann-Low formula together with Wick's theorem [20,5] to obtain the analytic expression for the relevant Green's functions. Afterwards we are then able to draw the corresponding Feynman diagrams.

As already mentioned in the first section of this chapter, we only use vertices of $\mathcal{L}^1_{HM}$ and $\mathcal{L}^2_{Mes}$. We are thus dealing with the following terms :

1.
$$\mathrm{tr}(\bar{B}i\gamma^{\mu}[D_{\mu}, B]\,) - M_0 \cdot \mathrm{tr}(\bar{B}B\,) \tag{4.12}$$

2.
$$\mathrm{tr}(\bar{B}i\gamma^{5}\gamma^{\mu}\{\Delta_{\mu}, B\}\,) \tag{4.13}$$

3.
$$\mathrm{tr}(\bar{B}i\gamma^{5}\gamma^{\mu}[\Delta_{\mu}, B]\,)\,. \tag{4.14}$$

What about the terms in $\mathcal{L}^2_{HM}$, that are proportional to the quark mass matrix $\mathcal{M}$ contained in the external field σ ? One can take two points of view depending on how one splits the Lagrangian into a free part $\mathcal{L}_0$ and an interacting part $\mathcal{L}_{int}$. Either we consider the quark mass term as part of $\mathcal{L}_0$, then the mass of the free particles includes the quark mass matrix $\mathcal{M}$, or we consider it as part of $\mathcal{L}_{int}$, then the mass of the free particles is given by their chiral mass M_0. In the following we include these terms in the free baryon Lagrangian.

Expanding the matrix $U = e^{i\Phi}$ and $u = e^{i\Phi/2}$ in powers of Φ, we find

$$\Gamma_{\mu} = -iv_{\mu} + \frac{i}{8}[\Phi, [\Phi, v_{\mu}]] + \frac{1}{8}[\Phi, \partial_{\mu}\Phi] + \cdots\,, \tag{4.15}$$

$$\Delta_{\mu} = \frac{i}{2}\partial_{\mu}\Phi - ia_{\mu} - \frac{1}{2}[\Phi, v_{\mu}] + \cdots\,. \tag{4.16}$$

For sake of completeness we also give the expansion of σ

$$\sigma = 2B_0 \cdot (\mathcal{M} - \frac{1}{8}\{\Phi, \{\Phi, \mathcal{M}\}\}) + \cdots\,, \tag{4.17}$$

where B_0 is a constant given in equation (3.52) in which the scalar field s has been replaced by $\mathcal{M}$. The expansion is given to the accuracy needed later on. It is now easy to derive the following relations:

$$\mathrm{tr}(\{\Delta_{\mu}, \Delta^{\mu}\}\,) = -\frac{1}{2}\mathrm{tr}(\partial_{\mu}\Phi\partial^{\mu}\Phi) + 2\mathrm{tr}(\partial_{\mu}\Phi a^{\mu}) - i\mathrm{tr}(\partial_{\mu}\Phi[\Phi, v^{\mu}]) \tag{4.18}$$

$$\begin{aligned}\mathrm{tr}(\bar{B}(i\gamma^{\mu}[D_{\mu}, B] - M_0 B)) &= \mathrm{tr}(i\gamma^{\mu}\partial_{\mu} - M_0)B) \\ &+ \mathrm{tr}(\bar{B}\gamma^{\mu}[v_{\mu}, B]) - \frac{1}{2}\mathrm{tr}(\bar{B}\gamma^{\mu}[[\frac{\Phi}{2}, [\frac{\Phi}{2}, v_{\mu}]], B]) + \frac{i}{2}\mathrm{tr}(\bar{B}\gamma^{\mu}[[\frac{\Phi}{2}, \partial_{\mu}\frac{\Phi}{2}], B])\end{aligned} \tag{4.19}$$

$$\mathrm{tr}(\bar{B}i\gamma^{5}\gamma^{\mu}\{\Delta_{\mu}, B\}) = -\mathrm{tr}(\bar{B}\gamma^{5}\gamma^{\mu}\{\partial_{\mu}\frac{\Phi}{2}, B\}) - i\mathrm{tr}(\bar{B}\gamma^{5}\gamma^{\mu}\{[\frac{\Phi}{2}, v_{\mu}], B\}) \tag{4.20}$$

$$\mathrm{tr}(\bar{B}i\gamma^{5}\gamma^{\mu}[\Delta_{\mu}, B]) = -\mathrm{tr}(\bar{B}\gamma^{5}\gamma^{\mu}[\partial_{\mu}\frac{\Phi}{2}, B]) - i\mathrm{tr}(\bar{B}\gamma^{5}\gamma^{\mu}[[\frac{\Phi}{2}, v_{\mu}], B])\,. \tag{4.21}$$

The first term on the right hand side of the equations(4.18,4.19) is part of the free Lagrangian $\mathcal{L}_0$; all other terms contribute to $\mathcal{L}_{int}$, which we write in the form

$$\mathcal{L}_{int} = \mathcal{L}^{int}_{Mes} + \mathcal{L}^{int}_{HM} \ . \tag{4.22}$$

For the interacting part of $\mathcal{L}_{Mes}$ and $\mathcal{L}_{HM}$ one obtains in the Gell–Mann basis

$$\mathcal{L}^{int}_{Mes} = F_0^2 \cdot f^{abc} \cdot \Phi^a \partial_\mu \Phi^b v^c_\mu \ , \tag{4.23}$$

$$\begin{aligned}\mathcal{L}^{int}_{HM} &= l^{abc} \cdot \bar{B}^a \gamma^5 \gamma^\mu B^b \cdot (\partial_\mu \Phi^c - f^{cde} \Phi^d v^e_\mu) \\ &-i f^{abc} \bar{B}^a \gamma^\mu B^b \cdot (v^c_\mu - \frac{1}{2} f^{cde} \Phi^d \partial_\mu \Phi^e + \frac{1}{2} f^{cdg} f^{gef} \Phi^d \Phi^e v^f_\mu) \ ,\end{aligned} \tag{4.24}$$

where

$$l^{abc} = D\ d^{abc} - iF\ f^{abc} \ . \tag{4.25}$$

We first look at the two point Green's function of the baryons:

$$G^{PQ}(x-y) \ = \ < 0|TB^P(x) \bar{B}^Q(y)\, e^{i\int d^4x_1 \mathcal{L}_{int}(x_1)}|0 > \ . \tag{4.26}$$

The Green's function in momentum space is defined by

$$(2\pi)^4 \delta^4(p-p') \tilde{G}^{PQ}(p) = \int d^4x \int d^4y\, e^{ipx} e^{-ip'y} G^{PQ}(x-y) \ . \tag{4.27}$$

After some calculation we obtain

$$\tilde{G}^{PQ}(p) = \frac{\not{p} + M_P}{p^2 - M_P^2 + i\epsilon} \cdot \delta^{PQ} + \frac{\not{p} + M_P}{p^2 - M_P^2 + i\epsilon} \cdot \frac{1}{F_0^2} L^{\ P}_{Q_1 Q_2} L^{\ Q}_{Q_1 Q_2} \cdot I_6 \cdot \frac{\not{p} + M_Q}{p^2 - M_Q^2 + i\epsilon} \ , \tag{4.28}$$

where

$$I_6 = i \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^5 \not{k} (\not{p} - \not{k} + M_{Q_1}) \gamma^5 \not{k}}{((p-k)^2 - M^2_{Q_1} + i\epsilon)(k^2 - m^2_{Q_2} + i\epsilon)} \ . \tag{4.29}$$

and where

$$L^R_{PQ} = D\ D^R_{PQ} - iF\ F^R_{PQ} \ . \tag{4.30}$$

In deriving this result all the external fields have been switched off except the quark mass matrix $\mathcal{M}$. Here and in the following the meson masses are denoted by m and the baryon masses by M. In the calculation we have used the representation of the free propagators in momentum space, given in appendix A.

Next we examine the three-point Green's function of two baryons and a vector current. It is given by the functional derivative of the baryon propagator with respect to the external vector field v

$$G^{RPQ}(x,y,z) \ = \ \frac{\delta}{\delta i v^R_\mu(z)} < 0|TB^P(x) \bar{B}^Q(y)\, e^{i\int d^4x_1 \mathcal{L}_{int}(x_1)}|0 > |_{v=0} \tag{4.31}$$

The expansion of the exponential gives rise to contributions to the Green's functions involving different powers of $\mathcal{L}_{int}$, i. e. , different number of vertices. We consider them separately :

1. The Fourier transform of $G^{RPQ}(x,y,z)$ is defined by

$$G^{RPQ}(p,p',q) = i^3 \int d^4x \int d^4y \int d^4z\, e^{ipx} e^{-ip'y} e^{iqz} G^{RPQ}(x,y,z) \,. \tag{4.32}$$

Extracting the overall momentum conservation, we write

$$G^{RPQ}(p,p',q) = i(2\pi)^4 \delta^4(p'-p-q) \tilde{G}^{RPQ}(p,p') \,. \tag{4.33}$$

For the contribution containing one vertex we obtain

$$\begin{aligned} \tilde{G}^{RPQ}(p,p') &= \frac{\not{p}+M_P}{p^2-M_P^2+i\epsilon} \gamma^\mu \frac{\not{p}'+M_Q}{p'^2-M_Q^2+i\epsilon} \cdot \\ &\quad (iF^Q_{RP} + \frac{i}{2F_0^2} F^Q_{Q_2P}\, F^{Q_3}_{Q_1Q_2}\, F^{Q_3}_{RQ_1} \cdot I_4) \,, \end{aligned} \tag{4.34}$$

where

$$I_4 = -i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_{Q_1}^2 + i\epsilon} \,. \tag{4.35}$$

$\tilde{G}^{RPQ}(p,p')$ has poles at $p^2 = M_P^2$ and $p'^2 = M_Q^2$. The baryon matrix element of the vector current can be read off from $\tilde{G}^{RPQ}(p,p')$ using the replacements

$$\frac{\not{p}+M_P}{p^2-M_P^2+i\epsilon} \to \bar{u}_P(p) \qquad \frac{\not{p}'+M_Q}{p'^2-M_Q^2+i\epsilon} \to u_Q(p') \,. \tag{4.36}$$

2. For the contribution with two vertices some lengthy algebra yields

$$\begin{aligned} &\tilde{G}^{RPQ}(p,p') \\ &= \frac{\not{p}+M_P}{p^2-M_P^2+i\epsilon} \Big(-\frac{i}{2F_0^2} F^Q_{Q_3P}\, F^{Q_3}_{Q_1Q_2}\, F^R_{Q_1Q_2} \cdot I_2^\mu \\ &+ \frac{i}{F_0^2} L^P_{Q_2Q_1}\, L^Q_{Q_2Q_3}\, F^{Q_3}_{RQ_1} \cdot I_5^\mu - \frac{i}{F_0^2} L^{Q_2}_{PQ_3}\, L^{Q_2}_{Q_1Q}\, F^{Q_3}_{RQ_1} \cdot I_7^\mu\Big) \\ &\cdot \frac{\not{p}'+M_Q}{p'^2-M_Q^2+i\epsilon} \,, \end{aligned} \tag{4.37}$$

where

$$I_2^\mu = i \int \frac{d^4k}{(2\pi)^4} \frac{(2\not{k}-\not{q})k^\mu}{((q-k)^2 - m_{Q_2}^2 + i\epsilon)(k^2 - m_{Q_1}^2 + i\epsilon)} \,, \tag{4.38}$$

$$I_5^\mu = -i \int \frac{d^4k}{(2\pi)^4} \gamma^5 \not{k} \frac{\not{p}-\not{k}+M_{Q_2}}{((p-k)^2 - M_{Q_2}^2 + i\epsilon)} \gamma^5 \gamma^\mu \frac{1}{(k^2 - m_{Q_1}^2 + i\epsilon)} \,, \tag{4.39}$$

$$I_7^\mu = -i \int \frac{d^4k}{(2\pi)^4} \gamma^5 \gamma^\mu \frac{\not{p}'-\not{k}+M_{Q_2}}{((p'-k)^2 - M_{Q_2}^2 + i\epsilon)} \gamma^5 \not{k} \frac{1}{(k^2 - m_{Q_1}^2 + i\epsilon)} \,. \tag{4.40}$$

3. After a lengthy calculation one obtains for the contribution with three vertices

$$
\begin{aligned}
&\tilde{G}^{RPQ}(p,p') \\
&= \frac{\not p + M_P}{p^2 - M_P^2 + i\epsilon} \cdot \Big(-\frac{i}{F_0^2} L^Q_{Q_3Q_1} L^{Q_3}_{PQ_2} F^R_{Q_2Q_1} \cdot I_1^\mu \\
&-\frac{i}{F_0^2} L^P_{Q_1Q_2} L^Q_{Q_3Q_1} F^{Q_3}_{RQ_2} \cdot \big(I_3^\mu - \gamma^\mu \frac{\not p' + M_{Q_2}}{p^2 - M_{Q_2}^2 + i\epsilon} I_6 - I_6 \frac{\not p + M_{Q_3}}{p^2 - M_{Q_3}^2 + i\epsilon}\gamma^\mu\big)\Big) \\
&\cdot \frac{\not p' + M_Q}{p'^2 - M_Q^2 + i\epsilon}, \qquad (4.41)
\end{aligned}
$$

where

$$
I_1^\mu = i \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^5(\not p - \not k)(\not k + M_{Q_3})\gamma^5(\not p' - \not k)\cdot(p'^\mu + p^\mu - 2k^\mu)}{((p-k)^2 - m_{Q_1}^2 + i\epsilon)(k^2 - M_{Q_3}^2 + i\epsilon)((p'-k)^2 - m_{Q_2}^2 + i\epsilon)}, \qquad (4.42)
$$

$$
I_3^\mu = -i \int \frac{d^4k}{(2\pi)^4} \frac{\gamma^5 \not k(\not p - \not k + M_{Q_2})\gamma^\mu(\not p' - \not k + M_{Q_3})\gamma^5 \not k}{((p-k)^2 - M_{Q_2}^2 + i\epsilon)(k^2 - m_{Q_1}^2 + i\epsilon)((p'-k)^2 - M_{Q_3}^2 + i\epsilon)} \qquad (4.43)
$$

and where I_6 is defined in equation (4.29).

All the integrals I_k introduced in this section are related to Feynman diagrams. The diagrams and their analytic expressions are listed in appendix C. The Feynman rules in momentum space can be read off from these expressions. Since we will not use them later on, we don't state them here. An detailed outline of the calculation is given in the appendices B and C of the thesis([21]). Here we only describe the main ideas.

4.4 Feynman parametrization

The evaluation of the integrals I_k is done in two steps: the momentum integration and the Feynman parameter integration. All the integrals I_k can be written as a combination of integrals of the form

$$
\begin{aligned}
F_1 &= \int \frac{d^4k}{(2\pi)^4} \frac{k^m}{k^2 - m_1^2 + i\epsilon} \\
F_2 &= \int \frac{d^4k}{(2\pi)^4} \frac{k^m}{((p_1-k)^2 - m_2^2 + i\epsilon)(k^2 - m_1^2 + i\epsilon)} \\
F_3 &= \int \frac{d^4k}{(2\pi)^4} \frac{k^m}{((p_2-k)^2 - m_3^2 + i\epsilon)((p_1-k)^2 - m_2^2 + i\epsilon)(k^2 - m_1^2 + i\epsilon)}, \qquad (4.44)
\end{aligned}
$$

where k^m denotes a tensor of rank m. To perform the momentum integration, we introduce Feynman parameter integrals as follows

$$\begin{aligned}&\frac{1}{A_1\cdots A_{n+1}}\\ &= \int_0^1 dx_1\cdots\int_0^1 dx_n \frac{n!\cdot x_1^0\cdots x_n^{n-1}}{[A_1x_1\cdots x_n + A_2(1-x_1)x_2\cdots x_n + \cdots + A_{n+1}(1-x_n)]^{n+1}}\,.\end{aligned} \tag{4.45}$$

With the help of (4.45) we can bring all momentum integrals to the form

$$F = \int \frac{d^4k}{(2\pi)^4}\frac{k^m}{[(p-k)^2 - M^2 + i\epsilon]^n}\,. \tag{4.46}$$

As is well known these integrals in general are divergent. They are only meaningful after regularization; we use dimensional regularization.

4.5 Dimensional regularization

Since we want to use dimensional regularization, we have to extend the Clifford algebra to higher dimensions. The formulae needed for the calculations are the following ones:

$$\begin{aligned} g^{\mu\nu} &= \begin{pmatrix} -1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{pmatrix} \\ \{\gamma^\mu,\gamma^\nu\} &= 2g^{\mu\nu} \\ \{\gamma_5,\gamma^\mu\} &= 0 \\ \gamma_5^2 &= \mathbb{1} \\ g^\mu_\mu &= d\,, \end{aligned} \tag{4.47}$$

where d denotes the number of space time dimensions. Usually the occurence of γ_5 leads to problems. This is not the case here, since no traces over γ matrices, involving γ_5, occur in our calculations.

We now investigate the integral

$$I = \int \frac{d^dk}{(2\pi)^d}\frac{1}{(k^2 - M^2 + i\epsilon)^n}\,. \tag{4.48}$$

In Minkowski space the integrand has poles at

$$k_0 = \pm(\sqrt{\vec{k}^2 + M^2} - i\epsilon)\,.$$

We perform a Wick rotation in the first component of k in such a way, that we don't interfere with the position of the poles. We thus substitute

$$k_0 = i\tilde{k}_0 \; .$$

The integral I then reads

$$\begin{aligned} I &= i\int \frac{d^d\tilde{k}}{(2\pi)^d}\frac{1}{(-\tilde{k}_0^2 - \vec{\tilde{k}}^2 - M^2 + i\epsilon)^n} \\ &= i\cdot(-1)^n \int \frac{d^d\tilde{k}}{(2\pi)^d}\frac{1}{(\tilde{k}^2 + M^2 - i\epsilon)^n} \,, \end{aligned} \tag{4.49}$$

where $\tilde{k}$ is a vector in a d-dimensional euclidean space. The integral can now be calculated quite easily (see [29]), obtaining

$$I = i(-1)^n \frac{1}{(4\pi)^{\frac{d}{2}}} \cdot \frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)} e^{(\frac{d}{2}-n)\ln(M^2 - i\epsilon)} \,. \tag{4.50}$$

Here $\Gamma(x)$ denotes the Gamma function. After a shift of variables $k = l + p$ in I, we can generate the integrals G_m

$$G_m = \int \frac{d^dl}{(2\pi)^d}\frac{l^{\mu_1}\dots l^{\mu_m}}{(l^2 - M^2 + i\epsilon)^n} \tag{4.51}$$

by applying successive derivatives with respect to p_μ on I and then putting $p = 0$. The G_m enable us to calculate all the momentum integrals of the last section in d dimensions.

4.6 Performing the momentum integration

As an example we perform the momentum integration for the integral I_4 explicitly.

$$\begin{aligned} I_4 &= -i(\mu^2)^{\frac{4-d}{2}} \int \frac{d^dk}{(2\pi)^d}\frac{1}{k^2 - m_1^2 + i\epsilon} \\ &= -\frac{1}{(4\pi)^2}(4\pi\mu^2)^{\frac{4-d}{2}}\frac{\Gamma(1-\frac{d}{2})}{\Gamma(1)} e^{(\frac{d}{2}-1)\ln(m_1^2 - i\epsilon)} \,. \end{aligned} \tag{4.52}$$

Expanding around $d = 4$, we find

$$I_4 = \frac{1}{(4\pi)^2} m_1^2 [\frac{1}{\eta} + 1 + \Gamma'(1) + \ln(\frac{4\pi\mu^2}{m_1^2})] \,, \tag{4.53}$$

where

$$\eta = 2 - \frac{d}{2} \,.$$

$\Gamma'(x)$ denotes the derivative of the Gamma function; terms of $O(\eta)$ have been neglected. For $d \neq 4$ a mass scale μ has been introduced; the argument of the logarithm then becomes dimensionless. The momentum integration for the other I_k can similarly be evaluated, but the involved algebra is lengthy. After the momentum integration has been carried out, it remains to perform the Feynman parameter integrations.

Even though simple in principle, the calculation of the integrals I_k is extremely tedious, involving a lot of technical details and leading to a rather monstrous result. In appendix C we therefore give only those results, which are used in the numerical analysis. For details of the parameter integration the reader is referred to appendix C of the thesis([21]).

4.7 Tree graphs

As already mentioned at the beginnning of the chapter, tree diagrams involving higher order vertices will also contribute to the matrix elements of the vector current. They have to be added to the loop contributions. Some of them are needed as counterterms for the divergent part of the loop integrals. All of them are proportional to a different coupling constant (low energy constant). Thus each of these contributions adds a new unknown to the low energy expansion. Although the low energy constants are in principle fixed by QCD, they cannot be determined using chiral symmetry alone. The general solution of the chiral Ward identities contains these constants as free parameters. In order to fix them one has to apply fits to experimental data or one has to use extra information, like the Zweig rule, to obtain estimates. In appendix B we have given a complete list of terms, which generate the effective baryon-meson Lagrangian up to $O(q^3)$. Looking at it we find the following contributions to the baryon matrix element of the vector current. At $O(q^2)$ we have

$$l_3 \cdot \mathrm{tr}(\bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}, B\}\,) + l_4 \cdot \mathrm{tr}(\bar{B}\sigma^{\mu\nu}[F_{\mu\nu}, B]\,). \tag{4.54}$$

This leads to the contribution

$$< p; P|V_R^\mu|p'; Q >= -4 \cdot (l_3\, D_{RP}^Q + il_4\, F_{RP}^Q) \cdot \bar{u}_P(p) i\sigma^{\mu\nu} q_\nu u_Q(p')\,, \tag{4.55}$$

with P, Q representing the outgoing and incoming baryon respectively and R the vector current. It constitutes the $SU(3)$ symmetric part of the magnetic moments of the baryons. The two constants l_3 and l_4 fix them completely. They will be renormalized by the one loop contributions.

The Lagrangian of $O(q^3)$ contains the terms

$$l_5 \cdot \mathrm{tr}(\bar{B}\gamma^\mu\{[D^\nu, F_{\mu\nu}], B\}\,) + l_6 \cdot \mathrm{tr}(\bar{B}\gamma^\mu[[D^\nu, F_{\mu\nu}], B]\,)\,. \tag{4.56}$$

The contribution to the vector current reads

$$< p; P|V_R^\mu|p'; Q >= 2 \cdot (l_5\, D_{RP}^Q + il_6\, F_{RP}^Q) \cdot \bar{u}_P(p) \left(q^2\gamma^\mu + (M_P - M_Q) q^\mu\right) u_Q(p')\,. \tag{4.57}$$

This term is needed as a counterterm for the one loop diagrams; otherwise it is not relevant for our numerical evaluation.

We are interested in the leading $SU(3)$ breaking effects of the magnetic moments. In addition to the loop contributions we find terms in the effective Lagrangian of $O(q^4)$, which lead to contributions linear in the quark mass. They will be taken into account in the numerical analysis. Eight independent terms occur in the effective Lagrangian. They are *not* listed in appendix B. As an example we give a typical term

$$\frac{1}{2} \cdot \mathrm{tr}(\bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}, [\sigma, B]\}\ ) + h.c. \quad . \tag{4.58}$$

The complete contribution to the vector current coming from these terms is given by

$$\begin{aligned}
< p; P|V_R^\mu|p'; Q >= (-2)N_{aP}^* N_{bQ} N_{cR}^* \cdot \bar{u}_P(p) i\sigma^{\mu\nu} q_\nu u_Q(p') \\
\cdot \Big[\mathcal{M}^d \Big(2k_1\delta^{ab}\delta^{cd} + k_2(\delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}) + k_3(d^{ace}d^{dbe} + d^{ade}d^{cbe}) \\
+ k_4 i(d^{ace}f^{dbe} + d^{cbe}f^{ade}) + k_5 i(d^{ade}f^{cbe} + d^{dbe}f^{ace}) + k_6 i^2(f^{dbe}f^{ace} + f^{ade}f^{cbe})\Big) \\
+ 6\mathcal{M}^0(k_7 d^{acb} + ik_8 f^{acb})\Big] \ .
\end{aligned} \tag{4.59}$$

Finally there are terms in the effective Lagrangian of $O(q^5)$, which contribute to the form factor F_1 at $q^2 = 0$; they are quadratic in the quark mass. We find three independent terms

$$\mathrm{tr}(\bar{B}\gamma^\mu\{[\sigma, [D_\mu, \sigma]], B\} + h.c. \tag{4.60}$$

$$\mathrm{tr}(\bar{B}\gamma^\mu[[\sigma, [D_\mu, \sigma]], B] + h.c. \tag{4.61}$$

$$\mathrm{tr}(\bar{B}[D_\mu, \sigma]) \cdot \gamma^\mu \cdot \mathrm{tr}(\sigma B) + h.c. \tag{4.62}$$

Chapter 5

Baryon Self Energy

5.1 Mass and wave function renormalization

For a complete calculation of S-matrix elements one has to take into account the renormalization of the vertices as well as the renormalization of the external legs. We are interested in the baryon matrix elements of the vector current. Thus in our case the external legs consist of two baryons and a quark current. The external field, which generates this current, has no dynamics. The external leg associated with it will therefore not be renormalized. The renormalization of the external baryons will be discussed in this chapter. In order to achieve this we look at the full baryon propagator. We switch off all the external fields except the scalar field σ and put it equal to the quark mass matrix $\mathcal{M}$.

We take into account the contributions coming from the tree diagrams up to $O(q^2)$ and the one loop diagram with vertices of $\mathcal{L}^1_{HM}$. In chapter 4 we have investigated the loop contribution. The terms in the Lagrangian, that are linear in the quark mass, have been chosen to be part of the free Lagrangian. They shift the mass of the baryons from their chiral value M_0 to a mass M_P. The one loop contribution is proportional to the integral I_6. Looking at this contribution we see that it is of the form

$$\tilde{G}_{PQ}(p) = \frac{1}{\not{p} - M_P + i\epsilon} \cdot \Sigma_{PQ}(p) \cdot \frac{1}{\not{p} - M_Q + i\epsilon} , \tag{5.1}$$

where

$$\Sigma_{PQ}(p) = \frac{1}{F_0^2} \sum_{Q_1,Q_2} L^P_{Q_1Q_2} \, L^Q_{Q_1Q_2} \cdot I_6(Q_1, Q_2) . \tag{5.2}$$

The matrix $\Sigma_{PQ}(p)$ is the baryon self energy. In general it will not be a diagonal matrix in flavour space. Only if one neglects $\Lambda - \Sigma^0$ mixing, the self energy becomes diagonal. Summing up an infinite number of self energy insertions, we obtain

$$\begin{aligned} \tilde{G}(p) &= \frac{1}{\not{p} - M + i\epsilon} \cdot \sum_{n=0}^{\infty} \left(\Sigma(p) \cdot \frac{1}{\not{p} - M + i\epsilon} \right)^n \\ &= (\not{p} - M - \Sigma(p) + i\epsilon)^{-1} . \end{aligned} \tag{5.3}$$

We are only interested in the limit $m_u = m_d$; we then have no mixing and the self energy as well as the propagator are diagonal. Decomposing the self energy in the form

$$\begin{aligned} \Sigma_{PQ}(p) &= A_{PQ}(p^2)\cdot \not p + B_{PQ}(p^2)\cdot \mathbf{1} \\ &= \frac{1}{F_0^2}\sum_{Q_1,Q_2} L^P_{Q_1Q_2}\, L^Q_{Q_1Q_2}\cdot \Big(A_8(p^2;Q_1,Q_2)\cdot \not p + B_8(p^2;Q_1,Q_2)\cdot \mathbf{1}\Big) \end{aligned} \tag{5.4}$$

we obtain the following equation for the physical mass M_{ph} of the baryons

$$\Big(1 - A(M_{ph}^2)\Big)\cdot M_{ph} = M + B(M_{ph}^2)\ . \tag{5.5}$$

Solving for M_{ph} we find

$$M_{ph} = M + B(M^2) + M\cdot A(M^2) + \cdots\ . \tag{5.6}$$

In this formula we have neglected two loop contributions and M_{ph} has been replaced by M in the one loop integrals. The functions A_8 and B_8 are given explicitly in appendix B. The divergent part of these functions contains a polynomial term of $O(m_q)$ as well as a term which does not vanish in the chiral limit. The chiral mass of the baryons M_0 and the low energy constants appearing in the effective Lagrangian $\mathcal{L}^2_{HM}$ are therefore renormalized.

To obtain an expression for the wave function renormalization Z of the baryons, we expand the inverse propagator around the physical mass M_{ph}

$$\tilde{G}^{-1}(p) = f(p) = (\not p - M_{ph})\cdot f'(M_{ph}) + \frac{1}{2}(\not p - M_{ph})^2\cdot f''(M_{ph}) + \cdots \quad . \tag{5.7}$$

We thus have

$$\tilde{G}(p) = \frac{Z}{\not p - M_{ph}} + \cdots \quad , \tag{5.8}$$

where the dots stand for non pole terms. Using (5.4,5.7,5.3) we obtain for Z

$$\begin{aligned} Z &= \left[1 - A(M_{ph}^2) - 2M_{ph}^2\frac{\partial A}{\partial p^2}(M_{ph}^2) - 2M_{ph}\frac{\partial B}{\partial p^2}(M_{ph}^2)\right]^{-1} \\ &= 1 + A(M^2) + 2M^2\frac{\partial A}{\partial p^2}(M^2) + 2M\frac{\partial B}{\partial p^2}(M^2) + \cdots \quad . \end{aligned} \tag{5.9}$$

In the last step we again have neglected two loop contributions and M_{ph} has been replaced by M in the one loop integrals. Both the nonanalytic part and the analytic part of Z are properly suppressed; thus the baryon wave function is not renormalized in the chiral limit.

5.2 Contribution of Z to S-matrix elements

The LSZ-formalism is the appropriate tool to extract S-matrix elements from the corresponding Green's functions. In momentum space the Green's functions have simple poles associated with an incoming and an outgoing particle. The residue of these poles is the product of the wave function renormalization of the particles and the S-matrix element. The renormalization of the external legs shifts the position of the pole and changes the wave function of the particle. In our case we are interested in the baryon matrix elements of the vector current. With the replacement

$$\frac{\not p + M}{p^2 - M^2 + i\epsilon} \rightarrow \frac{Z \cdot (\not p + M_{ph})}{p^2 - M_{ph}^2 + i\epsilon} \tag{5.10}$$

and the relation (see also appendix A)

$$Z \cdot (\not p + M_{ph}) = \sum_r Z^{1/2} u_{ph}(p;r) \cdot \bar{u}_{ph}(p;r) Z^{1/2} \tag{5.11}$$

we see, that in the S-matrix element the wave function renormalization leads to a factor $Z^{1/2}$ for every external baryon leg. Especially we find for the lowest order contribution to the vector current matrix element

$$< p; P|V_R^\mu|p'; Q >= \bar{u}_P(p)\gamma^\mu u_Q(p') \cdot Z_{PQ_1}^{1/2} \cdot iF_{RQ_1}^{Q_2} \cdot Z_{Q_2Q}^{1/2} \, . \tag{5.12}$$

In order to illustrate the need to take into account the wave function renormalization, we investigate a term in the effective Lagrangian, which is proportional to the equation of motion of the baryons. Such a term should not contribute to on-shell matrix elements. They are therefore omitted in the list of terms for $\mathcal{L}_{HM}$ given in appendix B. As an example we choose the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\mathrm{tr}(\bar{B}[\sigma, (i\gamma^\mu[D_\mu, B] - M_0 B)]\,) + h.c. \\ &= \bar{B}^a(i\gamma^\mu\partial_\mu - M_0)B^b \cdot 2if^{adb}\mathcal{M}^d \\ &\quad - \bar{B}^a\gamma^\mu B^b v_\mu^c \cdot (f^{ade}f^{cbe} + f^{ace}f^{dbe})\mathcal{M}^d \, . \end{aligned} \tag{5.13}$$

The contribution to the three-point Green's function reads

$$\begin{aligned} \tilde{G}^{RPQ} &= \frac{1}{\not p - M_0 + i\epsilon}\gamma^\mu \frac{1}{\not p' - M_0 + i\epsilon} \\ &\quad \cdot N_{Pa}^* N_{Qb} N_{Rc}^* i^2 (f^{ade}f^{cbe} + f^{ace}f^{dbe})\mathcal{M}^d \, , \end{aligned} \tag{5.14}$$

where the matrix N is defined in chapter 4 and is given in appendix A. If we ignore the wave function renormalization, we would thus find a contribution to the on-shell matrix element in disagreement with the statement above. Note that this also contradicts the Ademollo–Gatto theorem [1,26,10]; it states, that there is no contribution to the vector form factor F_1 at $q^2 = 0$, which is linear in the quark mass. Evaluating the corresponding contribution to the two-point Green's function, we obtain

$$Z_{PQ} = \delta_{PQ} - 2iN^*_{Pa}N_{Qb}f^{adb}\mathcal{M}^d \ . \tag{5.15}$$

Using equation(5.14) one can now easily show, that the contribution from Z and the one coming from $\tilde{G}^{RPQ}$ cancel each other. As a net result there is no contribution from the Lagrangian $\mathcal{L}$, given in equation (5.13), in agreement with the Ademollo–Gatto theorem.

Chapter 6

Combining the Diagrams

In chapter 4 we have loked at the contributions of the one loop diagrams. The one loop diagrams and their complete analytic expression H_k are given in appendix C. The explicit expression contains an integral I_k and a sum over flavour indices. In this chapter we examine the sum over the flavour group. As an illustration we look at H_3

$$H_3^\mu(R,P,Q) = \frac{i}{F_0^2} \sum_{Q_1,Q_2Q_3} F_{Q_2R}^{Q_3}\, L_{Q_1Q_2}^{P}\, L_{Q_3Q_1}^{Q} \cdot I_3^\mu(Q_1;Q_2,Q_3)\,. \tag{6.1}$$

The sums over the flavour indices cannot be done in a closed form, because the integral I_3 depends on these indices via the baryon and meson masses. If one evaluates the integral for the special case, when all the masses have their $SU(3)$ symmetric value, it becomes independent of the flavour indices. Using well known relations among the F- and D-symbols, it is then possible to carry out the flavour sums explicitly leading to terms proportional to the F- and D-symbols.

In general to each individual term of the flavour sum one can associate a set of physical particles running around in the loop. For example in the decay $\Xi^0 \to \Sigma^+$ we find the contribution

$$H_3^\mu(5,1,7) \cong \frac{i}{F_0^2} F_{45}^{8}\, L_{74}^{1}\, L_{87}^{7} \cdot I_3^\mu(7;4,8)\,. \tag{6.2}$$

The particles in the loop are the $\bar{K}^0$, the Λ and the p. On the other hand for fixed external legs the flavour sum gives us all allowed sets of particles in the loop, together with an appropriate weight factor (Clebsch–Gordan coefficient). An explicit evaluation of these sums has been done for all the relevant cases. An investigation of the results shows that different diagrams have similar sets of particles in the loop with similar Clebsch–Gordan coefficients. It is therefore possible to combine these diagrams; they then form three independent groups. In what follows we describe this in more detail.

The total contribution of the one loop diagrams and of the leading tree graph to the baryon matrix element of the vector current is given by

$$
\begin{aligned}
< p; P|V_R^\mu|p'; Q > \; = \; & \bar{u}_P(p) \cdot \Big(Z_{PQ_1}^{1/2} iF_{RQ_1}^{Q_2} Z_{Q_2Q}^{1/2} \cdot \gamma^\mu \\
& -iH_1^\mu + \frac{i}{2}H_2^\mu + iH_3^\mu + \frac{i}{2}H_4^\mu + iH_5^\mu - iH_7^\mu \Big) \cdot u_R(p') \, .
\end{aligned} \tag{6.3}
$$

The first term is the tree contribution including the wave function renormalization Z, discussed in the last chapter. The remaining terms are the analytic expressions of the one loop diagrams. The wave function renormalization Z has the form

$$
Z_{PQ} = \delta_{PQ} + \sum_{Q_1,Q_2} L_{Q_1Q_2}^P \, L_{Q_1Q_2}^Q \cdot I_8(Q_1;Q_2) \, , \tag{6.4}
$$

where I_8 is given by

$$
I_8(Q_1;Q_2) = A_8(M_{ph}^2) + 2M_{ph}^2 \cdot \frac{\partial A_8}{\partial p^2}(M_{ph}^2) + 2M_{ph} \cdot \frac{\partial B_8}{\partial p^2}(M_{ph}^2) \, . \tag{6.5}
$$

The functions A_8 and B_8 are defined in equation (5.4). Combining the appropriate contributions, the vector current matrix element can eventually be written in the form

$$
< p; P|V_R^\mu|p'; Q > = \bar{u}_P(p) \cdot (iF_{RP}^Q \gamma^\mu - iK_1^\mu + \frac{i}{2}K_2^\mu + iK_3^\mu) \cdot u_Q(p') \, . \tag{6.6}
$$

The functions K_1, K_2, K_3 contain the same sum over flavour indices as H_1, H_2, H_3 respectively; only the integrals I_k are replaced by the integrals J_k, defined by

$$
\begin{aligned}
J_1^\mu(Q_1,Q_2;Q_3) \; &= \; I_1^\mu(Q_1,Q_2;Q_3) + I_7^\mu(Q_1;Q_3|Q) + I_5^\mu(Q_2;Q_3|P) \\
& \quad + \frac{1}{2}I_8(Q_1;Q_3|Q) \cdot \gamma^\mu + \frac{1}{2}I_8(Q_2;Q_3|P) \cdot \gamma^\mu \\
J_2^\mu(Q_1,Q_2) \; &= \; I_2^\mu(Q_1,Q_2) + \frac{1}{2}I_4^\mu(Q_1) + \frac{1}{2}I_4^\mu(Q_2) \\
J_3^\mu(Q_1;Q_2,Q_3) \; &= \; I_3^\mu(Q_1;Q_2,Q_3) - \frac{1}{2}I_8(Q_1;Q_3|Q) \cdot \gamma^\mu - \frac{1}{2}I_8(Q_1;Q_2|P) \cdot \gamma^\mu \, .
\end{aligned} \tag{6.7}
$$

Thus we have reduced all the one loop contributions of the vector current to the three integrals J_k. All of them contain a piece proportional to $\gamma^\mu, i\sigma^{\mu\nu}q_\nu$ and q^μ, giving a contribution to F_1, F_2, F_3 respectively. We decompose the J_k in the following way

$$
J_k^\mu = J_k^{(1)}\gamma^\mu + J_k^{(2)} i\sigma^{\mu\nu}q_\nu + J_k^{(3)}q^\mu \, . \tag{6.8}
$$

Chapter 7

Numerical Analysis of the Form Factors

7.1 Analysis of $F_1(0)$

In this chapter we derive numerical results for the weak form factors F_1 and F_2 at $q^2 = 0$. We first consider F_1. In the contribution to F_1, the divergencies occurring in the one loop diagrams only cancel if the mass splitting of the baryons is disregarded. In the analysis of F_1 we therefore set all baryon masses equal. We illustrate the sensitivity of our results on the common mass value M_0 by considering two cases: $M_0 = 0.9$ Gev and $M_0 = 1.15$ Gev. The first value is an estimate of the mass of the baryon octet in the chiral limit, the second is the mean mass of the octet.

If the baryon masses are set equal, the Clebsch–Gordan sums simplify considerably; the form factor $F_1(0)$ can then be written in the form

$$
\begin{aligned}
F_1(\Xi^- \to \Sigma^0) &= \frac{1}{2}\Big([1 + \frac{3}{4}J^{(2)}_{K\eta} + \frac{3}{4}J^{(2)}_{K\pi}] \\
&\quad + \frac{1}{12}[(9F^2 + 18FD + 9D^2)J^{(1)}_{K\eta} + (9F^2 + 18FD + D^2)J^{(1)}_{K\pi}]\Big) \\
F_1(\Lambda \to p) &= \frac{1}{2}\sqrt{3}\Big([1 + \frac{3}{4}J^{(2)}_{K\eta} + \frac{3}{4}J^{(2)}_{K\pi}] \\
&\quad + \frac{1}{12}[(9F^2 + 6FD + D^2)J^{(1)}_{K\eta} + (9F^2 + 6FD + 9D^2)J^{(1)}_{K\pi}]\Big) \\
F_1(\Sigma^- \to n) &= \frac{1}{2}\sqrt{2}\Big([1 + \frac{3}{4}J^{(2)}_{K\eta} + \frac{3}{4}J^{(2)}_{K\pi}] \\
&\quad + \frac{1}{12}[(9F^2 - 18FD + 9D^2)J^{(1)}_{K\eta} + (9F^2 - 18FD + D^2)J^{(1)}_{K\pi}]\Big) \\
F_1(\Xi^- \to \Lambda) &= \frac{1}{2}\sqrt{3}\Big([1 + \frac{3}{4}J^{(2)}_{K\eta} + \frac{3}{4}J^{(2)}_{K\pi}] \\
&\quad + \frac{1}{12}[(9F^2 - 6FD + D^2)J^{(1)}_{K\eta} + (9F^2 - 6FD + 9D^2)J^{(1)}_{K\pi}]\Big)
\end{aligned}
$$

We have defined the abbreviations

$$J^{(1)}_{K\eta} = \frac{1}{(4\pi F_0)^2} \cdot J^{(1)}_1(m_K, m_\eta) \qquad J^{(1)}_{K\pi} = \frac{1}{(4\pi F_0)^2} \cdot J^{(1)}_1(m_K, m_\pi)$$

$$J^{(2)}_{K\eta} = \frac{1}{(4\pi F_0)^2} \cdot J^{(1)}_2(m_K, m_\eta) \qquad J^{(2)}_{K\pi} = \frac{1}{(4\pi F_0)^2} \cdot J^{(1)}_2(m_K, m_\pi) \, .$$

Here $J^{(1)}_k$ denotes the piece in J_k, which is proportional to γ^μ. The explicit formulae of the integrals $J^{(1)}_1$ and $J^{(1)}_2$ are given in appendix C.

The coefficients $J^{(1)}_2(m_K, m_\eta)$ and $J^{(1)}_2(m_K, m_\pi)$ stem from a purely mesonic loop (no internal baryon lines). Accordingly the explicit expression for J_2 only involves meson masses:

$$J^{(1)}_2 \quad = \quad -\frac{1}{4}(m_1^2 + m_2^2) + \frac{m_1^2 m_2^2}{2(m_1^2 - m_2^2)} \ln \frac{m_1^2}{m_2^2} \, . \tag{7.1}$$

Using $F_0 = 0.088$ Gev, we obtain

$$J^{(2)}_{K\eta} = -0.001 \qquad J^{(2)}_{K\pi} = -0.033 \, .$$

The net effect of the purely mesonic graphs is a universal reduction of the weak form factor $F_1(0)$ by 2.6%:

$$\frac{3}{4}(J^{(2)}_{K\eta} + J^{(2)}_{K\pi}) = -0.026 \, .$$

The same correction also occurs in the mesonic form factor $f^{K\pi}_+(0)$. In the case of $F_1(0)$, there is an additional contribution related to the triangle diagram with two mesonic and one baryonic internal line, described by $J^{(1)}_{K\eta}$ and $J^{(1)}_{K\pi}$.

At the beginning of our numerical analysis we study, if the leading terms in a chiral expansion of the integral J_1 are a good approximation of this function. When the baryons have a common mass M_0, the expansion of the integral $J^{(1)}_1$, given in appendix C, reduces to

$$J^{(1)}_1 = 3 \cdot J^{(1)}_2 + \frac{\pi}{2M_0}(m_1 - m_2)^2 \frac{m_1^2 + 3m_1 m_2 + m_2^2}{m_1 + m_2} + O(m^2_{quark}) \, . \tag{7.2}$$

The approximation includes contributions of $O(m_q \ln m_q, m_q^{3/2})$. For the leading term in the chiral expansion we have

$$J^{(1)}_{K\pi} = 3J^{(2)}_{K\pi} = -0.099$$

Numerical values of the functions $J^{(1)}_{K\eta}$ and $J^{(1)}_{K\pi}$ are listed in table 7.1 . The table shows that for the values of M_0 of interest, this term overestimates the size of the loop integral by a factor of order three. The chiral expansion converges very slowly.

The numerical results for the weak form factor F_1 are given in table 7.2. In the first column we have given the value of the form factor in the symmetry limit $m_u = m_d = m_s$. The other columns contain the results for F_1 normalized to F_1^{tree}. They include the $SU(3)$ breaking to one loop. M_0 denotes the common mass of the baryons. For the evaluation of the loop contributions we have chosen

$$F_0 = 0.088 \text{ GeV} \qquad F = 0.477 \qquad D = 0.755 \, .$$

The result for the form factor depends on the choice of the baryon mass M_0. We have studied various values of M_0 in order to estimate the induced error. When changing the value of M_0 from the mean mass 1.15 Gev to a chiral value 0.9 Gev, the form factor differs only by 0.5%. We have taken for F, D the best values of the $SU(3)$ symmetric fit, which are given by Bourquin et al. [6]. A variation of the F and D values leads to very small corrections in the asymmetry. The values obtained are strongly channel dependent, varying between 1.5% and 5%. The small value for the $\Sigma^- \to n$ transition is due to a positive contribution of the integral $J_1^{(1)}$, which partly balances the contribution of the integral $J_2^{(1)}$.

Donoghue et al. [14] have estimated the asymmetry of the form factor F_1 in a bag model calculation. They found a universal value of 0.987 for all $\Delta S = 1$ transitions. In contrast, our results vary up to 3% for different transitions. Only for the $\Sigma^- \to n$ transition we obtain a similar value for the asymmety.

The fact that the contributions generated by the one loop graphs are not well represented by the leading term in their expansion in powers of the quark mass presents us with the following problem. The motivation for considering the one loop graphs is the fact that they generate the leading contribution in the quark mass expansion of $F_1(0)$. The one loop graphs however also contain very substantial nonleading terms and we have included these contribution in the numerical results given in table 7.2 . Can we trust these nonleading terms or should we expect to find comparable contributions from two loop graphs or from higher order vertices in the effective Lagrangian? We cannot answer this question. It is conceivable that the asymmetries in the physical matrix elements are smaller than what we have found on the basis of the one loop calculation reported here. What this calculation does show, however, is that processes in which the W interacts with a virtual meson tend to generate surprisingly large and strongly asymmetric contributions and we see no reason why other processes should compensate this effect.

7.2 Analysis of $F_2(0)$

In this section we examine the form factor $F_2(0)$. As in the analysis of the form factor $F_1(0)$, we first neglect the mass splitting of the baryons. Later we consider the effect of the mass splitting. In our analysis contributions from tree graphs as well as one loop graphs are taken into account. For the electromagnetic transitions the tree contributions to the anomalous magnetic moments $F_2(0)$ are

$$
\begin{aligned}
F_2(\Sigma^+) &= -8M_0(\frac{l_3}{3}+l_4)-\frac{2}{9}\alpha((k_8+3k_1)+3k_7)\\
F_2(\Sigma^-) &= -8M_0(\frac{l_3}{3}-l_4)-\frac{2}{9}\alpha((k_8+3k_1)-3k_7)\\
F_2(\Sigma^0) &= -8M_0\frac{l_3}{3}-\frac{2}{9}\alpha(k_8+3k_1)\\
F_2(p) &= -8M_0(\frac{l_3}{3}+l_4)+\frac{1}{9}\alpha((k_8+3k_1)+3k_7-3k_6-9(k_5+k_1))\\
F_2(\Xi^-) &= -8M_0(\frac{l_3}{3}-l_4)+\frac{1}{9}\alpha((k_8+3k_1)-3k_7+3k_6-9(k_5+k_1))\\
F_2(n) &= -8M_0\frac{-2l_3}{3}-\frac{2}{9}\alpha((k_8+3k_1)-3k_6)\\
F_2(\Xi^0) &= -8M_0\frac{-2l_3}{3}-\frac{2}{9}\alpha((k_8+3k_1)+3k_6)\\
F_2(\Lambda) &= -8M_0\frac{-l_3}{3}-\frac{2}{9}\alpha((k_8+3k_1)+3k_2)\\
F_2(\Lambda\Sigma^0) &= -8M_0\sqrt{3}\frac{l_3}{3}-\frac{1}{9}\sqrt{3}\alpha k_2
\end{aligned}
$$

where

$$\alpha = 2\sqrt{3}\mathrm{tr}(\lambda_8\mathcal{M})\ .$$

In the chiral limit, $F_2(0)$ is completely determined by the two low energy constants l_3, l_4. Usually the magnetic moments of the nucleons are used to fix these constants. Then the magnetic moments of the hyperons can be expressed in terms of the anomalous magnetic moments of the nucleons. Coleman and Glashow [8] were the first who obtained this result. In addition we include the counterterms of $O(m_q)$ in our analysis. From the eight constants given in equation (4.59), k_7 and k_8 just redefine the constants l_3 and l_4; the other six low energy constants only appear in five independent combinations. Similarly we obtain the following tree contributions to the weak transition form factors:

$$
\begin{aligned}
F_2(\Sigma^0\Xi^-) &= -8M_0\frac{1}{2}\cdot(\frac{3l_3}{3}+l_4)-\frac{1}{12}\alpha((k_8+3k_1)+k_7-3k_6-3(k_5+k_1))\\
F_2(p\Lambda) &= -8M_0\frac{1}{2}\sqrt{3}\cdot(\frac{l_3}{3}+l_4)+\frac{1}{12}\sqrt{3}\alpha((k_8+3k_1)+3k_7-k_6-3(k_5+k_1)+4k_2)\\
F_2(n\Sigma^-) &= -8M_0\frac{1}{2}\sqrt{2}\cdot(\frac{-3l_3}{3}+l_4)+\frac{1}{12}\sqrt{2}\alpha((k_8+3k_1)-k_7+3k_6-3(k_5+k_1))\\
F_2(\Lambda\Xi^-) &= -8M_0\frac{1}{2}\sqrt{3}\cdot(\frac{-l_3}{3}+l_4)-\frac{1}{12}\sqrt{3}\alpha((k_8+3k_1)-3k_7+k_6-3(k_5+k_1)+4k_2)\\
F_2(pn) &= -8M_0\frac{1}{2}\sqrt{2}\cdot(\frac{3l_3}{3}+l_4)+\frac{1}{6}\sqrt{2}\alpha((k_8+3k_1)+k_7-3k_6-3(k_5+k_1))\\
F_2(\Lambda\Sigma^-) &= -8M_0\sqrt{3}\frac{l_3}{3}-\frac{1}{3}\sqrt{3}\alpha k_2\ .
\end{aligned}
$$

The loop contribution to the form factor $F_2(0)$ can be written in the form

$$\begin{aligned}
F_2(\Xi^- \to \Sigma^0) &= \frac{1}{24}[(9F^2+18FD+9D^2)J^{(1)}_{K\eta}+(9F^2+18FD+D^2)J^{(1)}_{K\pi}] \\
&\quad -\frac{1}{24}[(-6FD-2D^2)J^{(3)}_{\eta}+(6F^2-12FD+6D^2)J^{(3)}_{K} \\
&\quad +(12F^2-18FD+6D^2)J^{(3)}_{\pi}] \\
F_2(\Lambda \to p) &= \frac{1}{24}\sqrt{3}[(9F^2+6FD+D^2)J^{(1)}_{K\eta}+(9F^2+6FD+9D^2)J^{(1)}_{K\pi}] \\
&\quad -\frac{1}{24}[(-6FD+2D^2)J^{(3)}_{\eta}+(18F^2-12FD+2D^2)J^{(3)}_{K} \\
&\quad +(6FD+6D^2)J^{(3)}_{\pi}] \\
F_2(\Sigma^- \to n) &= \frac{1}{24}\sqrt{2}[(9F^2-18FD+9D^2)J^{(1)}_{K\eta}+(9F^2-18FD+D^2)J^{(1)}_{K\pi}] \\
&\quad -\frac{1}{24}[(6FD-2D^2)J^{(3)}_{\eta}+(6F^2+12FD+6D^2)J^{(3)}_{K} \\
&\quad +(12F^2+18FD+6D^2)J^{(3)}_{\pi}] \\
F_2(\Xi^- \to \Lambda) &= \frac{1}{24}\sqrt{3}[(9F^2-6FD+D^2)J^{(1)}_{K\eta}+(9F^2-6FD+9D^2)J^{(1)}_{K\pi}] \\
&\quad -\frac{1}{24}[(6FD+2D^2)J^{(3)}_{\eta}+(18F^2+12FD+2D^2)J^{(3)}_{K} \\
&\quad +(-6FD+6D^2)J^{(3)}_{\pi}]
\end{aligned}$$

Similar formulae can be obtained for the $\Delta S = 0$ transitions and for the electromagnetic transitions. We have defined the abbreviations

$$J^{(1)}_{K\eta} = \frac{1}{(4\pi F_0)^2}\cdot J^{(2)}_1(m_K, m_\eta) \qquad J^{(1)}_{K\pi} = \frac{1}{(4\pi F_0)^2}\cdot J^{(2)}_1(m_K, m_\pi)$$

$$J^{(3)}_{\eta} = \frac{1}{(4\pi F_0)^2}\cdot J^{(2)}_3(m_\eta) \qquad J^{(3)}_{K} = \frac{1}{(4\pi F_0)^2}\cdot J^{(2)}_3(m_K) \qquad J^{(3)}_{\pi} = \frac{1}{(4\pi F_0)^2}\cdot J^{(2)}_3(m_\pi)\,.$$

Here $J^{(2)}_k$ denotes the pieces in J_k, which is proportional to $\sigma^{\mu\nu}q_\nu$. The explicit formulae of the integral $J^{(2)}_1$ is given in appendix C.

We begin our analysis of F_2 with the study of the chiral expansion of the functions $J^{(2)}_1$ and $J^{(2)}_3$. From the formula of appendix C we obtain

$$\begin{aligned}
J^{(2)}_1 &= -4M_0^2 + \frac{8}{3}\pi M_0 \frac{m_1^2+m_1m_2+m_2^2}{m_1+m_2} \\
&\quad + \frac{4m_1^4}{m_1^2-m_2^2}\ln\frac{m_1^2}{M_0^2} - \frac{4m_2^4}{m_1^2-m_2^2}\ln\frac{m_2^2}{M_0^2} + O(m_{quark})\,.
\end{aligned}$$

For electromagnetic transitions, where $m_1 = m_2$, this reduces to

$$J^{(2)}_1 = -4M_0^2 + 4\pi M_0 m_1 + 8m_1^2\ln\frac{m_1^2}{M_0^2} + O(m_{quark})\,.$$

The leading terms in the expansion of the integral $J_3^{(2)}$ are

$$J_3^{(2)} = -4M_0^2 + 4m_1^2 \ln \frac{M_0^2}{m_1^2} + O(m_{quark}) \, ,$$

The approximation includes contributions of $O(m_q \ln m_q, m_q^{1/2})$. The leading term in this expansion is the square root singularity in the function $J_1^{(2)}$. It has already been calculated by Caldi and Pagels [7], using dispersion integral techniques. We can reproduce their result, if the Goldberger Treiman relation [10,26] is taken into account. Numerical values of the functions $J_{K\eta}^{(1)}$, $J_{K\pi}^{(1)}$ and $J_\eta^{(3)}$,$J_K^{(3)}$,$J_\pi^{(3)}$ are listed in table 7.3 . It turns out that the leading terms in a chiral expansion are not a good approximation of the loop integrals. In our analysis of $F_2(0)$ we thus do not use the approximated formulae.

The magnetic moment is related to the electromagnetic form factors $F_1(0)$ and $F_2(0)$ by

$$\mu^{PQ} = \frac{F_1^{PQ}(0) - F_2^{PQ}(0)}{M_P + M_Q} \, . \tag{7.3}$$

Data on magnetic moments are usually given in units of nuclear magnetons

$$\mu^{PQ} = \frac{\xi^{PQ}}{2M_{Proton}} \, .$$

Table 7.4 contains the magnetic moments of the baryons, which are experimentally well known.

The experimental data for the magnetic moments of the baryons are used to fix the low energy constants l_3,l_4 and k_i. We are then able to predict the magnetic moments for weak transitions. The way to use this information is however not unique. Either F_2 is taken to be $SU(3)$ symmetric, then physical masses have to be used in equation (7.3), or μ is symmetric, then M_P and M_Q have to be replaced by the common baryon mass M_0. We applied various fits to the low energy constants in order to estimate the induced error in the prediction of the weak magnetic moments. The results which we have obtained when loop contributions are neglected, are listed in table 7.5.

In the first two columns only a fit of l_3 and l_4 has been used to predict μ; the last two columns contain a fit with all low energy constants. The results show, that the uncertainties in the prediction of μ are reduced from about 20% with a fit of two parameters to about 10%, when all seven parameters are fitted. In the rest of our analysis we take the loop contributions into account. For the prediction of μ we use a fit of all seven parameters. Both procedures to fit the data have been applied. Table 7.6 contains the obtained results, when physical masses are used in equation (7.3). The first column contains the prediction for μ, when only tree contributions are used. In the second and the third column loop contributions with different values of the common baryon mass M_0 have been taken into account. In most transitions the predicted value of μ changes by about 20%.

Finally we use physical masses in the loop to predict μ. This is possible, because even with physical baryon masses, the loop integrals are free of ambiguities. The analytical expressions used in the numerical evaluation are extremely cumbersome and are therefore not given in this article. The results are listed in the last column of table 7.6. In this case, the tree level prediction is corrected for most of the transitions by about 10%. We are thus able to predict the weak magnetic moment and the weak form factor $F_2(0)$ within an accuracy of 10%.

	exact result		chiral approximation	
	$M_0 = 0.9$ Gev	$M_0 = 1.1511$ Gev	$M_0 = 0.9$ Gev	$M_0 = 1.1511$ Gev
$J^{(1)}_{K\eta}$	–0.0003	–0.0006	+0.003	+0.002
$J^{(1)}_{K\pi}$	–0.0312	–0.0400	+0.038	+0.008

Table 7.1: $F_1(0)$: Chiral approximation of the integral $J^{(1)}_1$

	$F_1^{tree}(0)$	$F_1^{loop}(0)/F_1^{tree}(0)$	
		$M_0 = 0.9$ Gev	$M_0 = 1.1511$ Gev
$(\Sigma^0\Xi^-)$	$\frac{1}{2}$	$1 - 0.050$	$1 - 0.057$
$(p\Lambda)$	$\frac{1}{2}\sqrt{3}$	$1 - 0.050$	$1 - 0.057$
$(n\Sigma^-)$	$\frac{1}{2}\sqrt{2}$	$1 - 0.016$	$1 - 0.013$
$(\Lambda\Xi^-)$	$\frac{1}{2}\sqrt{3}$	$1 - 0.039$	$1 - 0.043$

Table 7.2: Results for $F_1(0)$

	exact result		chiral approximation	
	$M_0 = 0.9$ Gev	$M_0 = 1.15$ Gev	$M_0 = 0.9$ Gev	$M_0 = 1.1511$ Gev
$J^{(1)}_{K\eta}$	–0.910	–1.779	+1.128	–0.088
$J^{(1)}_{K\pi}$	–1.229	–2.294	–0.429	–1.636
$J^{(3)}_{\eta}$	–3.142	–5.648	–1.675	–2.875
$J^{(3)}_{K}$	–3.321	–5.694	–1.691	–2.980
$J^{(3)}_{\pi}$	–2.957	–4.683	–2.418	–4.072

Table 7.3: $F_2(0)$: Chiral approximation of the integrals $J^{(2)}_1$ and $J^{(2)}_3$

	ξ^{PQ}
(Σ^+)	+2.379
(Σ^-)	–1.14
(p)	+2.793
(Ξ^-)	–0.69
(n)	–1.913
(Ξ^0)	–1.250
(Λ)	–0.613

Table 7.4: Data from experiment

	Fit of l_3, l_4		Fit of l_3, l_4, k_i	
	M_0	M_{phys}	M_0	M_{phys}
(Λ)	+0.510	+0.401	+0.330	+0.437
$(\Lambda\Sigma^0)$	+0.883	+0.718	+0.712	+0.738
$(\Sigma^0\Xi^-)$	+1.254	+0.936	+0.907	+1.004
$(p\Lambda)$	+1.289	+1.155	+1.081	+1.054
$(n\Sigma^-)$	-0.389	-0.355	-0.214	-0.276
$(\Lambda\Xi^-)$	+0.406	+0.313	+0.421	+0.357
(pn)	+1.773	+1.773	+1.773	+1.773
$(\Lambda\Sigma^-)$	+0.883	+0.716	+0.786	+0.773

Table 7.5: Results for μ^{PQ} from tree contributions

	tree level	1–loop		1–loop
		$M_0 = 0.9$	$M_0 = 1.15$	$M = M_{phys}$
(Σ^0)	+0.437	+0.437	+0.439	+0.431
$(\Lambda\Sigma^0)$	+0.738	+0.891	+0.925	+0.810
$(\Sigma^0\Xi^-)$	+1.004	+0.768	+0.770	+0.853
$(p\Lambda)$	+1.054	+0.776	+0.759	+0.894
$(n\Sigma^-)$	-0.276	-0.235	-0.226	-0.224
$(\Lambda\Xi^-)$	+0.357	+0.276	+0.264	+0.393
(pn)	+1.773	+1.630	+1.633	+1.827
$(\Lambda\Sigma^-)$	+0.773	+0.690	+0.692	+0.745

Table 7.6: Results for μ^{PQ} including one loop contributions

Summary and Conclusions

In the last years a considerable amount of data on the rates of the semileptonic hyperon decays have been accumulated. In particular a series of high statistic experiments has been performed at the SPS at CERN in 1982 [6]. The accuracy of the data is at the level of 1%-2%. In order to be able to use this information for a determination of the Kobayashi–Maskawa matrix element $|V_{us}|$, one has to know the vector form factor $F_1(0)$ very precisely. Until now most of the data on baryon decays have been analysed with the assumption of exact $SU(3)$ vector symmetry. With the precision of the new data it is necessary to take symmetry breaking effects into account.

As the main part of this article we have analysed the asymmetry of the vector form factor $F_1(0)$. In addition we have investigated the symmetry breaking for the magnetic form factor $F_2(0)$. We have calculated these form factors using chiral perturbation theory. The computation has been performed in the framework of an effective Lagrangian. The corrections to the $SU(3)$ values of the vector form factors have been obtained from a one loop calculation using vertices of $O(q)$. We have derived the leading terms in a chiral expansion of the form factors F_1 and F_2 of the weak and electromagnetic current. Caldi and Pagels [7] have calculated the leading correction to the $SU(3)$ value of the form factor $F_2(0)$; it is of $O(m_q^{1/2})$. We have reproduced this result using the Goldberger–Treiman relation. In addition we have computed the correction proportional to $m_q \ln m_q$. As already observed by Caldi and Pagels these corrections turn out to be large. Chiral perturbation theory is perfectly well defined. However, the leading terms in a chiral expansion of the loop integrals are *not* a good approximation of these integrals. The expansion can therefore not be used in the analysis of the one loop graphs.

The asymmetry of the weak form factor $F_1(0)$ generated by one loop graphs turns out to be substantial, varying between 1.5% to 5%. The errrors associated with these graphs are very small. The main uncertainty stems from higher order effects, including a contribution proportional to $(\hat{m} - m_s)^2$, which we did not attempt to estimate. In contrast to the results of bag model calculations, the asymmetry for $\Delta S = 1$ transitions strongly depends on the channel. The determination of the Kobayashi–Maskawa matrix element V_{us} from semileptonic hyperon decays is significantly affected by these asymmetries.

In the analysis of the weak form factor $F_2(0)$ the contribution of the counterterms proportional to m_q have been incorporated. The seven low energy constants have been fitted using the experimental data of the magnetic moments. It was then possible to predict the weak form factor $F_2(0)$ within an accuracy of 10%. Their contribution to the decay rates is kinematically suppressed. Our results may however be useful in an analysis of the angular distribution of semileptonic decays. By using only vertices of $O(q)$ in the loop contributions, the mass differences of the baryons have not been taken into account. They only show up in the effective Lagrangian of $O(q^2)$; at $O(q)$ all baryons have their common chiral mass M_0. The loop contributions affect the prediction for $F_2(0)$ by about 20%. In order to study the influence of the mass splitting, we have evaluated the loop integrals with physical baryon masses. It turns out that the mass differences reduce the effect of the loop contribution to about 10%. In a complete calculation one would have to compute all 1-loop contributions involving vertices of $O(q^2)$ and in addition the 2-loop contributions with vertices of $O(q)$. We have analysed the part of the 1-loop diagrams with higher order vertices, which is nonanalytic in the quark mass. The 2-loop contributions have not been investigated.

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Appendix A

Conventions and Notations

A.1 Metric, Dirac matrices and spinors

In this appendix we collect the basic conventions and notations used in the thesis.

1. Metric tensor:

$$g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \tag{A.1}$$

2. Scalar product:

$$a \cdot b = a_\mu b^\mu = g^{\mu\nu} a_\mu b_\nu \tag{A.2}$$

3. Levi–Civita tensor:

$$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} 1 & \text{if } (\mu,\nu,\alpha,\beta) \text{ is an even permutation of } (0,1,2,3) \\ -1 & \text{if it is an odd permutation} \\ 0 & \text{otherwise} \end{cases} \tag{A.3}$$

4. γ matrices:

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= 2g^{\mu\nu} \cdot \mathbf{1} \\ \gamma_0^\dagger &= \gamma_0 \\ \gamma_i^\dagger &= -\gamma_i \\ \gamma^5 = \gamma_5 &= i\gamma^0\gamma^1\gamma 2\gamma^3 \\ \gamma_5^\dagger &= \gamma_5 \\ \gamma_5^2 &= \mathbf{1} \\ \{\gamma^5, \gamma^\mu\} &= 0 \end{aligned} \tag{A.4}$$

5. Commutator of γ matrices:

$$
\begin{aligned}
\sigma^{\mu\nu} &= \frac{i}{2}[\gamma^\mu,\gamma^\nu] \\
[\gamma^5,\sigma^{\mu\nu}] &= 0 \\
\gamma^5\sigma^{\mu\nu} &= \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} \\
i\sigma^{\mu\nu} &= g^{\mu\nu} - \gamma^\mu\gamma^\nu = -g^{\mu\nu} + \gamma^\nu\gamma^\mu \\
\frac{1}{2}\{\sigma^{\mu\nu},\gamma^\alpha\} &= \epsilon^{\mu\nu\alpha\beta}\gamma^5\gamma_\beta \\
\frac{1}{2}[\sigma^{\mu\nu},\gamma^\alpha] &= -ig^{\mu\alpha}\gamma^\nu + ig^{\nu\alpha}\gamma^\mu
\end{aligned}
\tag{A.5}
$$

6. Hermitean conjugation:

$$
\begin{aligned}
\gamma^0\gamma^\mu\gamma^0 &= \gamma^{\mu\dagger} \\
\gamma^0\gamma^5\gamma^0 &= -\gamma^{5\dagger} \\
\gamma^0\gamma^5\gamma^\mu\gamma^0 &= (\gamma^5\gamma^\mu)^\dagger \\
\gamma^0\sigma^{\mu\nu}\gamma^0 &= (\sigma^{\mu\nu})^\dagger
\end{aligned}
\tag{A.6}
$$

7. Charge conjugation:

$$
\begin{aligned}
C\gamma^\mu C^{-1} &= -\gamma^{\mu T} \\
C\gamma^5 C^{-1} &= \gamma^{5T} \\
C\gamma^5\gamma^\mu C^{-1} &= (\gamma^5\gamma^\mu)^T \\
C\sigma^{\mu\nu}C^{-1} &= -(\sigma^{\mu\nu})^T
\end{aligned}
\tag{A.7}
$$

8. Dirac equation:

$$(\not p - M)u(p;r) = 0 \tag{A.8}$$

$$\bar u(p;r)(\not p - M) = 0 \tag{A.9}$$

9. Normalization of spinors:

$$\bar u(p;r)u(p;s) = 2M\delta_{rs} \tag{A.10}$$

$$u^\dagger(p;r)u(p;s) = 2E_p\delta_{rs}\ , \tag{A.11}$$

where $E_p = \sqrt{\vec p^2 + M^2}$

10. Projection operator:

$$\sum_r u(p;r)\otimes \bar u(p;r) = \not p + M \tag{A.12}$$

11. Gordon identities:

$$\bar{u}(p;r)\gamma^{\mu}u(p';s) = \frac{1}{2M}\bar{u}(p;r)\left(r^{\mu} - i\sigma^{\mu\nu}q_{\nu}\right)u(p';s) \tag{A.13}$$

$$\bar{u}(p;r)\gamma^{5}\gamma^{\mu}u(p';s) = \frac{1}{2M}\bar{u}(p;r)\gamma^{5}\left(q^{\mu} - i\sigma^{\mu\nu}r_{\nu}\right)u(p';s)\ , \tag{A.14}$$

where $r_{\mu} = p'_{\mu} + p_{\mu}$ and $q_{\mu} = p'_{\mu} - p_{\mu}$

12. Normalization of fermion states:

$$< p;r|p';s >= 2E_p(2\pi)^3\delta^3(\vec{p}-\vec{p}')\delta_{rs} \tag{A.15}$$

13. Invariant measure:

$$\int \frac{d^3p}{(2\pi)^3 2E_p} \tag{A.16}$$

14. Fourier expansion of spinor fields:

$$B(x) = \sum_r \int \frac{d^3p}{(2\pi)^3 2E_p}\left(u(p;r)b(p;r)e^{-ipx} + v(p;r)d^{\dagger}(p;r)e^{ipx}\right) \tag{A.17}$$

Nearly all conventions are taken from the textbook of Itzykson and Zuber [20]; exceptions are the normalization of spinors, the normalization of fermion states and related formulae. In $u(p;r)$ the argument r denotes the polarization of the particle; $\vec{p}$ is the three momentum of the particle. In all objects carrying a flavour index, it has not been indicated explicitly.

A.2 Baryon fields and meson fields

$$B = \frac{1}{\sqrt{2}}B_A\tilde{\lambda}^A\ ;\ \bar{B} = \frac{1}{\sqrt{2}}\bar{B}_A\tilde{\lambda}^{\dagger A}\ ;\ \Phi = \Phi_A\tilde{\lambda}^A \qquad A = 1,\cdots,8\ , \tag{A.18}$$

where $\tilde{\lambda}_A$ are the generators of $SU(3)$ in the physical basis. They are related to the Gell–Mann matrices λ_a by

$$\tilde{\lambda}_A = \sum_a N_{Aa}\lambda_a \qquad a, A = 1,\cdots,8 \tag{A.19}$$

$$\lambda^a = \sum_A N^*_{Aa}\tilde{\lambda}^A = \sum_A N_{Aa}\tilde{\lambda}^{\dagger A}\ , \tag{A.20}$$

The non zero elements of the matrix N, N_{Aa}, are [16,17]:

$$\begin{aligned} N_{11} = N_{44} = N_{66} = N_{77} &= \frac{-1}{\sqrt{2}} \\ N_{21} = N_{54} &= \frac{1}{\sqrt{2}} \end{aligned}$$

$$
\begin{aligned}
N_{12} = N_{22} = N_{45} = N_{55} = N_{67} &= \frac{-i}{\sqrt{2}} \\
N_{77} &= \frac{i}{\sqrt{2}} \\
N_{33} = N_{88} &= \cos(\epsilon) \\
N_{83} &= -\sin(\epsilon) \\
N_{38} &= \sin(\epsilon) \\
\tan(2\epsilon) &= \frac{\mathrm{tr}(\lambda_3 \mathcal{M})}{\mathrm{tr}(\lambda_8 \mathcal{M})},
\end{aligned}
$$

where

$$
\mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}
$$

is the quark mass matrix. Neglecting $SU(2)$ splitting, the mixing angle ϵ is zero. The matrix N is a unitary 8×8 matrix. We thus have

$$
\begin{aligned}
\sum_a N^*_{Aa} N_{Ba} &= \delta^{AB} \\
\sum_A N^*_{Aa} N_{Ab} &= \delta^{ab}\,.
\end{aligned}
\tag{A.21}
$$

The components of the fields in the two bases are related by

$$
\begin{aligned}
\Phi_a &= \sum_A N_{Aa} \Phi_A = \sum_A N^*_{Aa} \Phi^*_A \\
B_a &= \sum_A N_{Aa} B_A \\
\bar{B}_a &= \sum_A N^*_{Aa} \bar{B}_A \\
V^\mu_a &= \sum_A N_{Aa} \tilde{V}^\mu_A \\
\tilde{V}^\mu_a &= \sum_a N^*_{Aa} V^\mu_A\,.
\end{aligned}
$$

In particular we find

$$
\begin{aligned}
\tilde{V}^\mu_5 &= \frac{1}{\sqrt{2}}(V^\mu_4 + iV^\mu_5) \\
\tilde{V}^\mu_2 &= \frac{1}{\sqrt{2}}(V^\mu_1 + iV^\mu_2)\,.
\end{aligned}
$$

The components of the fields in the *physical basis* B_A, Φ_A can be identified with the physical particles:

$$
B_1 = \Sigma^+\,, B_2 = \Sigma^-\,, B_3 = \Sigma^0\,, B_4 = p
$$

$$B_5 = \Xi^- \,, B_6 = n \,, B_7 = \Xi^0 \,, B_8 = \Lambda$$
$$\Phi_1 = \pi^+ \,, \Phi_2 = \pi^- \,, \Phi_3 = \pi^0 \,, \Phi_4 = K^+$$
$$\Phi_5 = K^- \,, \Phi_6 = K^0 \,, \Phi_7 = \bar{K}^0 \,, \Phi_8 = \eta \,.$$

The baryon matrix B and the meson matrix Φ then take the form

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & -\Sigma^+ & -p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & -n \\ \Xi^- & -\Xi^0 & \frac{-2}{\sqrt{6}}\Lambda \end{pmatrix} \tag{A.22}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{2}}\pi^0 & -\pi^+ & -K^+ \\ \pi^- & \frac{1}{\sqrt{6}}\eta - \frac{1}{\sqrt{2}}\pi^0 & -K^0 \\ K^- & -\bar{K}^0 & \frac{-2}{\sqrt{6}}\eta \,. \end{pmatrix} \tag{A.23}$$

Let $b^\dagger_\Lambda(p;r)$ be the creation operator in the Fourier expansion of $B_8(x) = \Lambda(x)$; it generates the physical state $|\Lambda >$

$$|\Lambda; p; r >= b^\dagger_\Lambda(p;r)|0>$$

with similar relations for the other states. The choice of the matrix N agrees with the phase conventions of Condon-Shortley and De Swart [13].

A.3 Free meson and baryon propagators

The free propagators in the physical basis are

$$\begin{aligned} < 0|T\Phi^A_{in}(x)\Phi^{*B}_{in}(y)|0> &= \frac{i}{F_0^2}\Delta^A(x-y)\cdot\delta^{AB} \\ \Delta^A(x-y) &= \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_A^2 + i\epsilon}\, e^{-ik(x-y)} \end{aligned} \tag{A.24}$$

$$\begin{aligned} < 0|TB^A_{in}(x)\bar{B}^{*B}_{in}(y)|0> &= iS^A(x-y)\cdot\delta^{AB} \\ S^A(x-y) &= \int \frac{d^4k}{(2\pi)^4}\frac{\not{k} + M_A}{k^2 - M_A^2 + i\epsilon}\, e^{-ik(x-y)}\,, \end{aligned} \tag{A.25}$$

where F_0 is the meson decay constant, m_A is the meson mass and M_A is the baryon mass.

Appendix B

Effective Baryon-Meson Lagrangian

In chapter 3 we have given a recipe to construct the effective baryon-meson Lagrangian. It turned out that the most general term with two baryon fields and n other fields has two distinct forms. Either the baryon fields are embraced by one trace over the flavour group or they are standing under separate traces. In order to check the charge conjugation property of a term we use (anti-) commutators of the fields instead of products. We define a quantity c by

$$c = c_1 + \cdots + c_n + c_\Gamma ,$$

where c_k denotes the charge conjugation of the field A_k. For the class of terms with one trace only we then find the following combinanations of the fields $A_1, \ldots, A_n$:

1. for $n = 1$
   If c is even one can build two independent terms with even charge conjugation
   $$\{A_1, B\} \qquad [A_1, B]$$
   The first term is an abbreviation for the expression
   $$\frac{1}{2}\left(\mathrm{tr}(\bar{B}\Gamma[A_1, B]) + \mathrm{tr}(\bar{B}\Gamma[A_1, B])^c\right)$$
   If c is odd there is no such term.

2. for $n = 2$
   If c is even we have the four independent terms
   $$\{A_1, \{A_2, B\}\} \qquad [A_1, \{A_2, B\}]$$
   $$\{A_1, [A_2, B]\} \qquad [A_1, [A_2, B]]$$
   If c is odd we find
   $$\{[A_1, A_2], B\} \qquad [[A_1, A_2], B]$$

3. for $n = 3$
   For even c we can build the following 12 terms

$$\{A_1,\{A_2,\{A_3,B\}\}\} \qquad \{A_1,[A_2,\{A_3,B\}]\} \qquad [A_1,\{A_2,\{A_3,B\}\}]$$
$$\{A_1,\{A_2,[A_3,B]\}\} \qquad [A_1,[A_2,\{A_3,B\}]] \qquad [A_1,\{A_2,[A_3,B]\}]$$
$$\{A_1,[A_2,[A_3,B]]\} \qquad [A_1,[A_2,[A_3,B]]] \qquad \{[A_1,[A_2,A_3]],B\}$$
$$[[A_1,[A_2,A_3]],B] \qquad \{[A_2,[A_1,A_3]],B\} \qquad [[A_2,[A_1,A_3]],B]$$

   If c is odd the independent terms are

$$\{[A_1,A_2],\{A_3,B\}\} \qquad \{[A_1,A_2],[A_3,B]\} \qquad [[A_1,A_2],\{A_3,B\}]$$
$$[[A_1,A_2],[A_3,B]] \qquad \{[A_1,A_3],\{A_2,B\}\} \qquad \{[A_1,A_3],[A_2,B]\}$$
$$[[A_1,A_3],\{A_2,B\}] \qquad [[A_1,A_3],[A_2,B]] \qquad \{[A_2,A_3],\{A_1,B\}\}$$
$$\{[A_2,A_3],[A_1,B]\} \qquad [[A_2,A_3],\{A_1,B\}] \qquad [[A_2,A_3],[A_1,B]]$$

A further reduction of terms might still be possible, using identities in the Clifford algebra. Those which are used in chapter 3 are

$$i\sigma^{\alpha\beta} = g^{\alpha\beta}\mathbf{1} - \gamma^\alpha\gamma^\beta = -g^{\alpha\beta}\mathbf{1} + \gamma^\beta\gamma^\alpha$$
$$\frac{1}{2}\{\sigma^{\alpha\beta},\gamma^\mu\} = \frac{i}{4}\{[\gamma^\alpha,\gamma^\beta],\gamma^\mu\} = \epsilon^{\alpha\beta\mu\nu}\gamma^5\gamma_\nu$$
$$\frac{1}{2}[\sigma^{\alpha\beta},\gamma^\mu] = \frac{i}{4}[[\gamma^\alpha,\gamma^\beta],\gamma^\mu] = -i(g^{\alpha\mu}\gamma^\beta - g^{\beta\mu}\gamma^\alpha)\,. \tag{B.1}$$

In the following list we give all allowed terms of the effective baryon-meson Lagrangian up to $O(q^3)$; terms proportional to the equation of motion have been omitted.

B.1 Terms of $O(q)$

$$\mathrm{tr}(\bar{B}i\gamma^\mu[D_\mu,B] - M_0\bar{B}B) \qquad \mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu,B\})$$

B.2 Terms of $O(q^2)$

$\mathrm{tr}(\bar{B}\{\sigma,B\})$ $\quad$ $\mathrm{tr}(\bar{B}\sigma^{\mu\nu}\{F^+_{\mu\nu},B\})$
$\mathrm{tr}(\bar{B}i\sigma^{\mu\nu}\{[\Delta_\mu,\Delta_\nu],B\})$ $\quad$ $\mathrm{tr}(\bar{B}\{\Delta_\mu,\{\Delta^\mu,B\}\})$ $\quad$ $\mathrm{tr}(\bar{B}i\gamma^\mu\{\Delta_\mu,\{\Delta_\nu,[D^\nu,B]\}\})$
$\mathrm{tr}(\bar{B}B)\cdot\mathrm{tr}(\sigma)$ $\quad$ $\mathrm{tr}(\bar{B}B)\cdot\mathrm{tr}(\Delta_\mu,\Delta^\mu)$ $\quad$ $\mathrm{tr}(\bar{B}i\gamma^\mu[D^\nu,B])\cdot\mathrm{tr}(\Delta_\mu\Delta_\nu)$
$\mathrm{tr}(\bar{B}\Delta^\mu)\cdot\mathrm{tr}(\Delta_\mu B)$ $\quad$ $\mathrm{tr}(\bar{B}\Delta_\mu)\cdot i\sigma^{\mu\nu}\cdot\mathrm{tr}(\Delta_\nu B)$
$\mathrm{tr}(\bar{B}\Delta_\mu)\cdot i\gamma^\mu\cdot\mathrm{tr}(\Delta_\nu[D^\nu,B])$ $\quad$ $\mathrm{tr}(\bar{B}\Delta_\mu)\cdot\mathrm{tr}(\Delta_\nu[D^\mu,[D^\nu,B]])$

B.3 Terms of $O(q^3)$

$\mathrm{tr}(\bar{B}\gamma^5\{\varrho, B\})$ $\mathrm{tr}(\bar{B}\gamma^5\{[D^\mu, \Delta_\mu], B\})$ $\mathrm{tr}(\bar{B}\gamma^\mu\{[D^\nu, F^+_{\mu\nu}], B\})$
$\mathrm{tr}(\bar{B}\gamma^5\gamma^\mu\{[D^\nu, F^-_{\mu\nu}], B\})$ $\mathrm{tr}(\bar{B}i\gamma^\mu\{[\Delta_\mu, [D^\nu, \Delta_\nu]], B\})$
$\mathrm{tr}(\bar{B}i\gamma^\mu\{[\Delta_\nu, [D^\nu, \Delta_\mu]], B\})$ $\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu, \{\sigma, B\}\})$
$\mathrm{tr}(\bar{B}i\gamma^\mu\{\Delta_\mu, \{\varrho, B\}\})$ $\mathrm{tr}(\bar{B}\gamma^5\gamma^\mu\{[\Delta^\nu, F^+_{\mu\nu}], B\})$
$\mathrm{tr}(\bar{B}i\gamma_\mu\{\Delta_\nu, \{F^+_{\alpha\beta}, B\}\}) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}\gamma^\mu\{[\Delta^\nu, F^-_{\mu\nu}], B\})$
$\mathrm{tr}(\bar{B}i\gamma^5\gamma_\mu\{\Delta_\nu, \{F^-_{\alpha\beta}, B\}\}) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu, \{\Delta^\nu, \{\Delta_\nu, B\}\}\})$
$\mathrm{tr}(\bar{B}\gamma_\mu\{[\Delta_\alpha, \Delta_\beta], \{\Delta_\nu, B\}\}) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}\gamma^5\sigma^{\mu\nu}\{[\Delta_\alpha, F^+_{\mu\nu}], [D^\alpha, B]\})$
$\mathrm{tr}(\bar{B}\sigma^{\mu\nu}\{\Delta_\alpha, \{F^-_{\mu\nu}, [D^\alpha, B]\}\})$ $\mathrm{tr}(\bar{B}i\gamma^\mu\{[\Delta_\alpha, [D_\mu, \Delta_\beta]], [D^\alpha, [D^\beta, B]]\})$
$\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{[\Delta_\alpha, \{\Delta_\mu, \{\Delta_\beta, [D^\alpha, [D^\beta, B]]\}\}\})$
$\mathrm{tr}(\bar{B}\gamma^5 B) \cdot \mathrm{tr}(\varrho)$ $\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu, B\}) \cdot \mathrm{tr}(\sigma)$
$\mathrm{tr}(\bar{B}i\gamma_\mu B) \cdot \mathrm{tr}(\Delta_\nu F^+_{\alpha\beta}) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}i\gamma^5\gamma_\mu B) \cdot \mathrm{tr}(\Delta_\nu F^-_{\alpha\beta}) \cdot \epsilon^{\mu\nu\alpha\beta}$
$\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu, B\}) \cdot \mathrm{tr}(\Delta_\nu\Delta^\nu)$ $\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\nu, B\}) \cdot \mathrm{tr}(\Delta_\mu\Delta^\nu)$
$\mathrm{tr}(\bar{B}\gamma_\mu B) \cdot \mathrm{tr}([\Delta_\alpha, \Delta_\beta]\Delta_\nu) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}\sigma^{\mu\nu}[D^\alpha, B]) \cdot \mathrm{tr}(\Delta_\alpha F^-_{\mu\nu})$
$\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu\{\Delta_\alpha, [D^\alpha, [D^\beta, B]]\}) \cdot \mathrm{tr}(\Delta_\mu\Delta_\beta)$
$\mathrm{tr}(\bar{B}i\gamma^5\gamma^\mu[D^\alpha, [D^\beta, B]]) \cdot \mathrm{tr}(\Delta_\mu\{\Delta_\alpha, \Delta_\beta\})$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma^\mu \cdot \mathrm{tr}(\sigma B)$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^\mu \cdot \mathrm{tr}(\varrho B)$
$\mathrm{tr}(\bar{B}\Delta^\mu) \cdot \gamma^\nu \cdot \mathrm{tr}(F^-_{\mu\nu} B)$ $\mathrm{tr}(\bar{B}\Delta^\mu) \cdot \gamma^5\gamma^\nu \cdot \mathrm{tr}(F^+_{\mu\nu} B)$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma_\nu \cdot \mathrm{tr}(F^-_{\alpha\beta} B) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma_\nu \cdot \mathrm{tr}(F^+_{\alpha\beta} B) \cdot \epsilon^{\mu\nu\alpha\beta}$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma_\nu \cdot \mathrm{tr}([\Delta_\alpha, \Delta_\beta]B) \cdot \epsilon^{\mu\nu\alpha\beta}$ $\mathrm{tr}(\bar{B}\Delta^\mu) \cdot i\gamma^5\gamma^\nu \cdot \mathrm{tr}([\Delta_\mu, \Delta_\nu]B)$
$\mathrm{tr}(\bar{B}\Delta^\mu) \cdot i\gamma^5\gamma^\nu \cdot \mathrm{tr}(\{\Delta_\mu, \Delta_\nu\}B)$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma^\mu \cdot \mathrm{tr}(\{\Delta^\nu, \Delta_\nu\}B)$
$\mathrm{tr}(\bar{B}\Delta^\mu) \cdot \gamma^\nu \cdot \mathrm{tr}([D_\mu, \Delta_\nu]B)$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^\mu \cdot \mathrm{tr}([D^\nu, \Delta_\nu]B)$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i \cdot \mathrm{tr}(\varrho[D^\mu, B])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\sigma^{\mu\nu} \cdot \mathrm{tr}([\Delta_\nu, \Delta_\alpha][D^\alpha, B])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\sigma^{\nu\alpha} \cdot \mathrm{tr}([\Delta_\nu, \Delta_\alpha][D^\mu, B])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^5\sigma^{\mu\nu} \cdot \mathrm{tr}(\{\Delta_\nu, \Delta_\alpha\}[D^\alpha, B])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^5\sigma^{\mu\nu} \cdot \mathrm{tr}(F^+_{\nu\alpha}[D^\alpha, B])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^5\sigma^{\nu\alpha} \cdot \mathrm{tr}(F^+_{\nu\alpha}[D^\mu, B])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \sigma^{\mu\nu} \cdot \mathrm{tr}(F^-_{\nu\alpha}[D^\alpha, B])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \sigma^{\nu\alpha} \cdot \mathrm{tr}(F^-_{\nu\alpha}[D^\mu, B])$
$\mathrm{tr}(\bar{B}\Delta^\mu) \cdot \cdot \mathrm{tr}(F^-_{\mu\nu}[D^\nu, B])$ $\mathrm{tr}(\bar{B}\Delta^\mu) \cdot i \cdot \mathrm{tr}([D_\mu, \Delta_\nu][D^\nu, B])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i \cdot \mathrm{tr}([D^\nu, \Delta_\nu][D^\mu, B])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\sigma^{\mu\nu} \cdot \mathrm{tr}([D_\nu, \Delta_\alpha][D^\alpha, B])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^5\gamma^\alpha \cdot \mathrm{tr}([\Delta_\nu, \Delta_\alpha][D^\nu, [D^\mu, B]])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma^\alpha \cdot \mathrm{tr}(\{\Delta_\nu, \Delta_\alpha\}[D^\nu, [D^\mu, B]])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma^\mu \cdot \mathrm{tr}(\{\Delta_\nu, \Delta_\alpha\}[D^\nu, [D^\alpha, B]])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^5\gamma^\alpha \cdot \mathrm{tr}(F^+_{\nu\alpha}[D^\nu, [D^\mu, B]])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i\gamma^\alpha \cdot \mathrm{tr}(F^-_{\nu\alpha}[D^\nu, [D^\mu, B]])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^\alpha \cdot \mathrm{tr}([D_\nu, \Delta_\alpha][D^\nu, [D^\mu, B]])$
$\mathrm{tr}(\bar{B}\Delta_\mu) \cdot \gamma^\mu \cdot \mathrm{tr}([D_\nu, \Delta_\alpha][D^\nu, [D^\alpha, B]])$ $\mathrm{tr}(\bar{B}\Delta_\mu) \cdot i \cdot \mathrm{tr}([D_\nu, \Delta_\alpha][D^\mu, [D^\nu, [D^\alpha, B]]])$

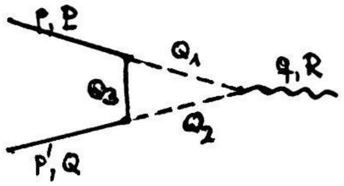
In the list a term X is an abbreviation for $\frac{1}{2}(\mathrm{tr}(X) + \mathrm{tr}(X)^c)$; a term in the list represents all combinations of (anti-) commutators allowed by charge conjugation, as discussed above.

Appendix C

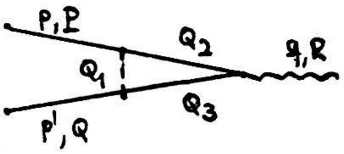
Feynman Diagrams and Explicit Formulae

C.1 Feynman diagrams

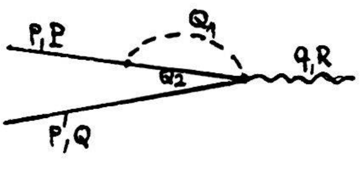
In chapter 4 we stated the analytical expressions of the one loop diagrams for the vector current matrix elements of the baryons. In the following we list these diagrams and the corresponding analytical expressions; the integrals I_k are given in chapter 4.



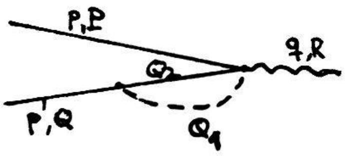
$$= H_1^\mu = \bar{u}(p) \cdot \frac{-i}{F_0^2} L^B_{A_3A_1}\, L^{A_3}_{AA_2}\, F^C_{A_2A_1} \cdot I_1^\mu \cdot u(p') \qquad \text{(C.1)}$$



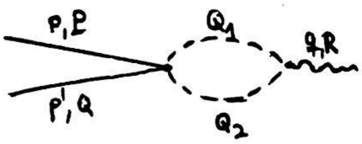
$$= H_3^\mu = \bar{u}(p) \cdot \frac{-i}{F_0^2} L^A_{A_1A_2}\, L^B_{A_3A_1}\, F^{A_3}_{CA_2} \cdot I_3^\mu \cdot u(p') \qquad \text{(C.2)}$$



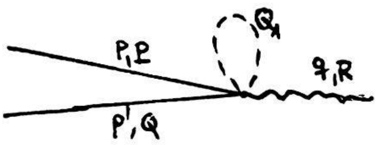
$$= H_5^\mu = \bar{u}(p) \cdot \frac{i}{F_0^2} L^A_{A_2A_1}\, L^B_{A_2A_3}\, F^{A_3}_{CA_1} \cdot I_5^\mu \cdot u(p') \qquad \text{(C.3)}$$



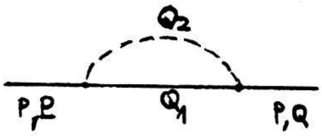
$$= H_7^\mu = \bar{u}(p) \cdot \frac{-i}{F_0^2} L^{A_2}_{AA_3}\, L^{A_2}_{A_1B}\, F^{A_3}_{CA_1} \cdot I_7^\mu \cdot u(p') \qquad \text{(C.4)}$$



$$= H_2^\mu = \bar{u}(p) \cdot \frac{-i}{2F_0^2} F^B_{A_3 A}\, F^{A_3}_{A_1 A_2}\, F^C_{A_1 A_2} \cdot I_2^\mu \cdot u(p') \tag{C.5}$$



$$= H_4 = \bar{u}(p) \cdot \frac{i}{2F_0^2} F^B_{A_2 A}\, F^{A_3}_{A_1 A_2}\, F^{A_3}_{C A_1} \cdot I_4 \cdot u(p') \tag{C.6}$$



$$= H_6 = \frac{1}{F_0^2} L_{A_1 A_2}^{\;A} L_{A_1 A_2}^{\;B} \cdot I_6 \tag{C.7}$$

Solid lines represent baryons, dashed lines mesons and wavy lines the vector current.

C.2 Formulae for A_8 and B_8

In chapter 5 we have parametrized the self energy of the baryons in terms of the functions A_8 and B_8. We write the results in the form

$$Q_1 \cdot \tilde{\kappa} + Q_2 + Q_3 \ln\left(\frac{a_0 + a_1 + a_2}{a_0}\right)$$
$$+\, Q_4 (a_1^2 - 4a_0 a_2)^{1/2} \ln\left(\frac{2a_0 + a_1 + (a_1^2 - 4a_0a_2)^{1/2}}{2a_0 + a_1 - (a_1^2 - 4a_0a_2)^{1/2}}\right) .$$

$$\tilde{\kappa} = \frac{1}{\eta} + \Gamma'(1) + \frac{1}{2}\ln\left(\frac{4\pi\mu^2}{a_0}\right) + \frac{1}{2}\ln\left(\frac{4\pi\mu^2}{a_0 + a_1 + a_2}\right)$$
$$a_0 = m_2^2 \qquad a_1 = M_1^2 - m_2^2 - p^2 \qquad a_2 = p^2 .$$

We introduced the abbreviation m_2 for m_{Q_2} and M_1 for M_{Q_1}. The coefficient functions Q_k are given by

1. for A_8

$$\begin{aligned}
Q_1 &= \frac{1}{2}(2m_2^2 + 3M_1^2 - p^2) \\
Q_2 &= \frac{1}{2p^2}(-M_1^4 + 5M_1^2p^2 + M_1^2m_2^2 - 2p^4 + 3p^2m_2^2) \\
Q_3 &= \frac{1}{4p^4}(M_1^6 - 3M_1^4p^2 - 2M_1^4m_2^2 + M_1^2m_2^4 + p^2m_2^4) \\
Q_4 &= \frac{1}{4p^4}(-M_1^4 + 2M_1^2p^2 + M_1^2m_2^2 - p^4 + p^2m_2^2)
\end{aligned}$$

2. for B_8

$$\begin{aligned}
Q_1 &= M_1^2 + m_2^2 \\
Q_2 &= M_1^2 + 2m_2^2 \\
Q_3 &= \frac{1}{2p^2}(-M_1^2 p^2 - M_1^2 m_2^2 + m_2^4) \\
Q_4 &= \frac{m_2^2}{2p^2}
\end{aligned}$$

C.3 Formulae for the J_k

The functions J_k have been calculated for physical masses in the loop and on the external legs, and for arbitrary q^2. Even for $q^2 = 0$ the obtained formulae are extremely long, filling several pages. In our opinion it is therefore not worthwhile to present them here. For the interested reader the general results are available. In the following we list approximations for the formulae of J_k which are used in the numerical analysis of the form factors.

The function J_2 only contributes to the form factor F_1; for $q^2 = 0$ it reads

$$J_2^{(1)} = -\frac{1}{4}(m_1^2 + m_2^2) + \frac{m_1^2 m_2^2}{2(m_1^2 - m_2^2)} \ln \frac{m_1^2}{m_2^2} . \tag{C.8}$$

A representation of the function J_1 at $q^2 = 0$ can be given in terms of rational functions H_{k0} of the physical masses. When all baryons have a common mass M_0, it has the form

$$\begin{aligned}
J_1(m_1^2, m_2^2; M_0) &= H_{20} + H_{50} \cdot \ln(\frac{m_2^2}{M_0^2}) + H_{40} \cdot \ln(\frac{m_1^2}{M_0^2}) \\
&+ H_{60} \cdot (-2)\nu^{1/2}(m_1^2) \cdot \arctan(\frac{\nu^{1/2}(m_1^2)}{m_1^2}) + H_{70} \cdot (-2)\nu^{1/2}(m_2^2) \cdot \arctan(\frac{\nu^{1/2}(m_2^2)}{m_2^2}) ,
\end{aligned} \tag{C.9}$$

where

$$\nu(x) = 4M_0^2 x - x^2 .$$

The function J_1 contains a piece $J_1^{(1)}$ proportional to γ^μ, which contributes to the form factor $F_1(0)$ and a piece $J_1^{(2)}$ proportional to $\sigma^{\mu\nu} q_\nu$ contributing to the form factor $F_2(0)$. We give the coefficient functions for $J_1^{(1)}$ and $J_1^{(2)}$ separately. For $J_1^{(1)}(m_1^2, m_2^2; M_0^2)$ the coefficient functions H_{k0} are:

$$\begin{aligned}
H_{20} &= \frac{1}{4}(-3m_1^2 - 3m_2^2) \\
H_{40} &= \frac{-m_1^6 + 3m_1^4 m_2^2 - 3m_1^2 m_2^2 M_0^2}{-2m_1^2 M_0^2 + 2m_2^2 M_0^2}
\end{aligned}$$

$$
\begin{aligned}
H_{50} &= \frac{3m_1^2m_2^2(-m_2^2+M_0^2)+m_2^6}{-2m_1^2M_0^2+2m_2^2M_0^2} \\
H_{60} &= \frac{m_1^6+m_1^4(-3m_2^2-2M_0^2)+10m_1^2m_2^2M_0^2}{-2m_1^4M_0^2+2m_1^2M_0^2(m_2^2+4M_0^2)-8m_2^2M_0^4} \\
H_{70} &= \frac{m_1^2m_2^2(3m_2^2-10M_0^2)+m_2^4(-m_2^2+2M_0^2)}{2m_1^2M_0^2(-m_2^2+4M_0^2)+2m_2^2M_0^2(m_2^2-4M_0^2)}
\end{aligned}
$$

For $J_1^{(2)}(m_1^2, m_2^2; M_0^2)$ the coefficient functions H_{k0} are:

$$
\begin{aligned}
H_{20} &= \frac{1}{3}(8m_1^2+4(2m_2^2-3M_0^2)) \\
H_{40} &= \frac{4m_1^6-12m_1^4M_0^2}{-3m_1^2M_0^2+3m_2^2M_0^2} \\
H_{50} &= \frac{4m_2^4(-m_2^2+3M_0^2)}{-3m_1^2M_0^2+3m_2^2M_0^2} \\
H_{60} &= \frac{-4m_1^4+4m_1^2M_0^2}{-3m_1^2M_0^2+3m_2^2M_0^2} \\
H_{70} &= \frac{4m_2^2(m_2^2-M_0^2)}{-3m_1^2M_0^2+3m_2^2M_0^2}
\end{aligned}
$$