What symmetry is broken in the superconductor - normal phase transition?

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Abstract. We show that the 2D superconducting - normal phase transition is due to spontaneous breaking of magnetic flux symmetry generated by $\Phi = \int d^2xB(x)$. In the normal phase this symmetry is spontaneously broken with massless photon as a corresponding Goldstone boson. In the superconducting phase the symmetry is unbroken and the magnetic flux annihilates the vacuum which expresses the essence of the Meissner effect. We explicitly construct the pertinent gauge invariant order parameter which is the operator creating Abrikosov vortices. Its vacuum expectation value vanishes in the superconducting ground state, while is finite in the vacuum of the normal phase.

Second order phase transitions are frequently, if not always, associated with spontaneous breakdown of a global symmetry. It is then possible to find a corresponding order parameter which vanishes in the disordered phase and is nonzero in the ordered phase.

At the first glance the superconductor - normal phase transition is an important exception. Colloquially it is said sometimes that the electric charge $U_e(1)$ symmetry is broken in superconductor with nonvanishing "order parameter" $\Delta \equiv < \psi_\uparrow \psi_\downarrow >$. This however, should not be taken literally. If a continuous symmetry is spontaneously broken, the Goldstone theorem ensures the existence of massless excitations. On the other hand, the physical spectrum of a superconductor does not contain any such excitation. Also the " order parameter" Δ does not vanish *only when there is no coupling to electromagnetism.* Consequently in the complete theory including electromagnetism, $U_e(1)$ is not broken spontaneously. The interpretation of this is that the "would be Goldstone boson" is eaten by the photon which acquires a mass (finite penetration depth) via Anderson - Higgs mechanism.

This mechanism ("local gauge symmetry breaking") however lacks description in physical terms. Indeed, according to the very general Elitzur's theorem, local symmetry can be never broken and a non gauge invariant quantity never acquires nonzero vacuum expectation value (VEV).

Thus neither global electric charge $U_e(1)$ nor local gauge symmetry is broken at the superconductor - normal phase transition. However, as we show in this paper, such a symmetry does exist. It is identified here as a continuous symmetry generated by magnetic flux.

The flux symmetry is generated by the full magnetic flux penetrating the plane

$$\Phi = \int B(x)d^2x \tag{1}$$

The continuity equation for the corresponding current $\rho_\Phi \equiv B$; $j^i_\Phi \equiv -\epsilon^{ij}E_j$, $i, j, = 1,2$ is the homogeneous Maxwell equations of electrodynamics.

We show that the mode of realization of this $U_\Phi(1)$ symmetry distinguishes between the superconducting and the normal phases. In the normal phase $U_\Phi(1)$ is spontaneously broken with massless photon as a Goldstone boson. The superconducting vacuum is invariant under the action of $U_\Phi(1)$, the symmetry is unbroken.

The order parameter V(x) is defined by $[\Phi, V(x)] = -gV(x)$. It is therefore a creation operator of magnetic vortex of flux g. Its explicit form is

$$V(x) = C \exp ig \int d^2y[a_i(x-y)E_i(y) + eb(x-y)J_0(y)] \tag{2}$$

where J_0 is electric charge density. A convenient choice for the coefficient functions is

$$a_i(x) = \frac{1}{2\pi}\epsilon_{ij}\frac{x_j}{x^2}, \quad b(x) = \frac{1}{2\pi}\Theta(x) \tag{3}$$

where $\Theta(x)$ is an angle between the vector x_i and the $\hat{x}_1$ axis, $0 \leq \Theta < 2\pi$. The locality of $V(x)$ with respect to gauge invariant electric current density requires quantization of the eigenvalue g, $g = \frac{2\pi n}{e}$. This coincides with the usual quantization of magnetic flux of the Abrikosov vortices.

In the normal phase Φ does not annihilate vacuum but rather creates a soft photon: $\Phi|0>= L^{1/2}|\omega(1/L)>$, where L is an infrared cutoff. The states $e^{i\zeta\Phi}|0>$ are degenerate and the flux symmetry breaks down, producing the Goldstone boson - massless photon. The expectation value of V in this phase does not vanish [1].

In the superconductor due to finite penetration depth Φ annihilates the vacuum. Consistently the VEV of the order parameter V vanishes [1]. A single Abrikosov vortex is an eigenstate of the conserved "charge" Φ. The energy gap Δ_Φ between the vacuum and the lightest flux carrying state - the vortex - is finite. This expresses the essence of Meissner effect. Application of small external magnetic field amounts to the addition to the Hamiltonian of a perturbation ΔH, which also commutes with Φ. The perturbed vacuum acquires an admixture of excited states, but since ΔH commutes with Φ it cannot contain states with nonzero flux. Consequently infinitesimal magnetic field cannot penetrate the sample. However when the external field is large enough this perturbative argument is not valid any more, and eventually a fluxon is produced. The required magnetic energy must be larger than the energy gap in the fluxon channel. Therefore H_{c_1} is determined by Δ_Φ. In this picture it is clear that creation of fluxons is the only way the magnetic field can penetrate the sample since any mixed state should be an eigenstate of Φ.

We stress that in terms of symmetries of the *physical* Hilbert space the symmetry breaking pattern is reversed compared to the aforementioned colloquial terminology: in the superconducting phase no symmetry is broken while the normal state breaks the flux symmetry.

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References

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