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Observing an Ising transition in Josephson-junction arrays

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Abstract. *Using dynamic simulations we study electric and magnetic ac response of a Josephson-junction array near an Ising transition. The transition is visible in the in-phase component but we see no significant increase in the dissipation.*

We consider a rectangular lattice of superconducting islands, each of which is connected to nearest neighbors by Josephson junctions. The island centered at e.g. $\mathbf{r}$ is described by one parameter $\varphi(\mathbf{r},t)$. The time evolution of the phase difference across junction $< \mathbf{r},\mathbf{r}' >$ is governed by:

$$\dot{\varphi}(\mathbf{r}) - \dot{\varphi}(\mathbf{r}') = i(\mathbf{r},\mathbf{r}') - \sin(\varphi(\mathbf{r}) - \varphi(\mathbf{r}') - 2\pi A(\mathbf{r},\mathbf{r}')) - i_F(\mathbf{r},\mathbf{r}',t) \quad (1)$$

Time is measured in units of the reciprocal characteristic frequency $1/\omega_c = \hbar/(2eI_cR_n)$, with a single-junction critical current I_c, and normal-state resistance R_n. We allow for a magnetic field perpendicular to the plane of the array, and this enters through the variables $A(\mathbf{r},\mathbf{r}') = \frac{1}{\phi_0}\int_{\mathbf{r}}^{\mathbf{r}'} \mathbf{A}\cdot d\boldsymbol{\ell}$, with vector potential $\mathbf{A}$. The physical quantity is f, the magnetic flux piercing a unit cell of the array, in units of the elementary flux quantum ϕ_0. The total current carried by the junction is $i(\mathbf{r},\mathbf{r}')$, in units of I_c. For non-zero temperature there is a fluctuating current i_F, which we take to be a white-noise source. The Langevin equations for different junctions are coupled because the total current entering each island has to be zero at each instant of time. This coupled set, describing the *full dynamics* of junction arrays, can be simulated very efficiently using realistic boyundary conditions, with an algorithm introduced earlier [1].

We apply the algorithm to the special case of a generalization of the fully frustrated Josephson-junction array [2]. In this model the horizontal junctions, $< \mathbf{r},\mathbf{r}+\hat{\mathbf{x}} >$, alternately have critical current I_c and ηI_c, and all vertical junctions have critical current I_c. There is a magnetic field with $f = \frac{1}{2}$. The fully frustrated array, $\eta = 1$, shows one phase transition with both Ising-like and Kosterlitz-Thouless (KT)-like features [3]. For $\eta \neq 1$ the transition splits in an Ising and a KT transition [2], and in an earlier paper we explained the origin of this transition using a Coulomb-gas analysis [4]. In particular, we showed that the Ising transition originates from small vortex dipoles, formed at the horizontal junctions with the smaller critical current. One can build an array with $\eta \neq 1$ by varying the surface area of selected junctions. This provides the possibility to measure the Ising transition in experiments on junction arrays.

One manifestation of the transition is as a kink in the helicity modulus [4], and as a first attempt one can try to observe the transition by measuring the non-linear resistance of arrays. However, preliminary results show that this is hard in practice as the effect is very subtle [5]. Therefore we consider the response of the array with $\eta = 0.5$ to a small oscillating signal, which may be either a magnetic field or a current. First we concentrate on the measurements with a magnetic field, because they have been performed very successfully to observe the KT transition [6]. Results of extensive simulations in a 16×16 array showed that the effect of the edges is large if the field is uniform over the array. Consequently, we consider here a 32×32 array with the field amplitude decreasing smoothly to zero near the edges. In Fig. 1a we show the results of the in-phase, and out-of-phase component of the magnetization μ of the current distribution, χ'_μ and χ''_μ, respectively. The amplitude of oscillations in f in the center of the array is $5 \cdot 10^{-3}$. From Monte Carlo simulations we know that the Ising transition occurs at approximately $T = 0.18$. The results clearly show a kink in χ'_μ at the Ising point. Because the oscillating field induces complicated current patterns, χ'_μ is a linear combination of all components of the helicity modulus $\Gamma(\mathbf{k})$. A priori it is not clear that this will also show the kink, present in the helicity modulus in the x-direction which is $\lim_{k_x \to 0} \Gamma(k_x, k_y = 0)$. Unfortunately the Ising transition does not clearly show in χ''_μ. The same lack of features in χ''_μ is observed at higher frequencies. Next we try to gain more insight by considering the response of the average phase difference Φ across the array in the x-direction to an oscillating current in the same direction. The lattice size is $L = 16$ and

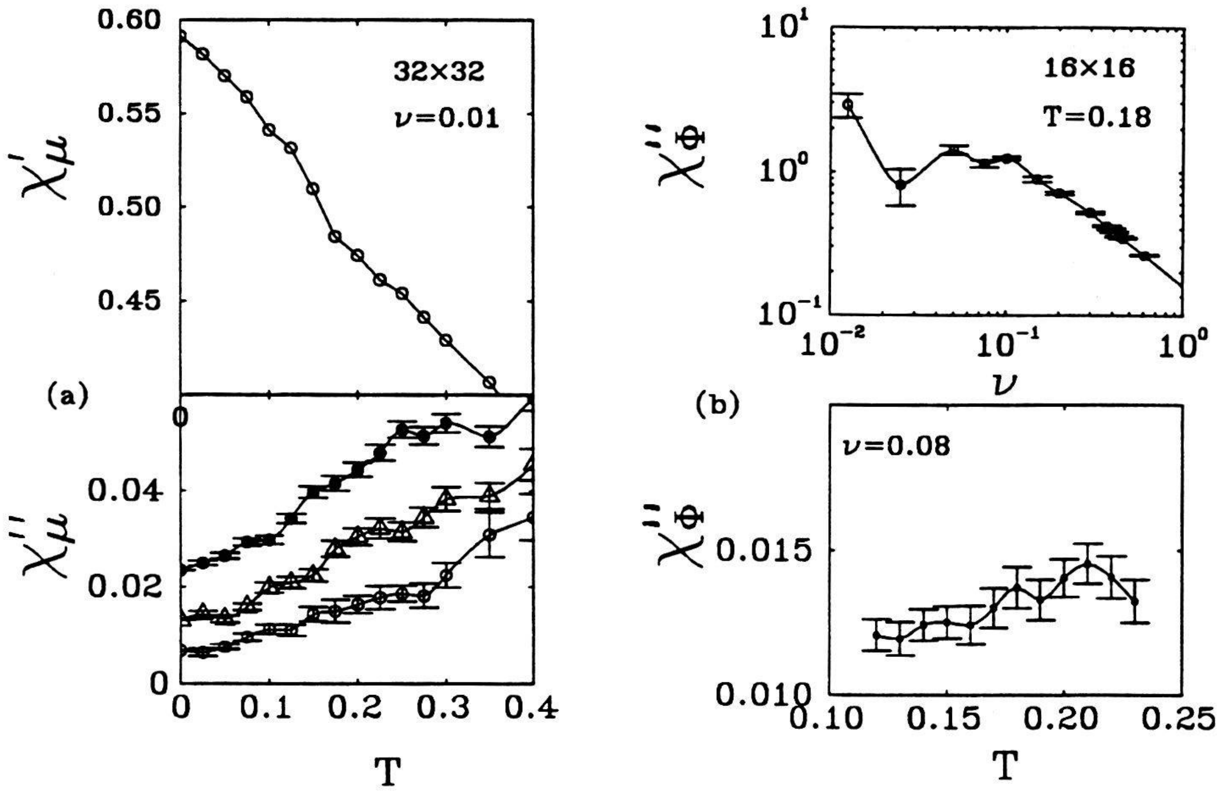

Figure 1: *The response to a small oscillating magnetic field (a) for frequencies* $\nu = 0.01$ *(○), 0.02 (△), and 0.04 (•), in units of* $\omega_c/2\pi$. *The normalization factor is* $f \cdot (\partial\mu/\partial f)|_{f=0}$, *where* μ *is the magnetization of the ground state current distribution. Figure b gives the response of* Φ *to an oscillating current in the x-direction.*

the amplitude of the total external current is $0.01 I_c \times 16$. In Fig. 1b we show the frequency dependence of χ''_Φ at $T = 0.18$. The frequency dependence is very smooth and the shape does not have strongly temperature dependent features. We also plot χ''_Φ at $2\pi\nu = 0.08\omega_c$. Clearly, this measuring technique does not improve the results drastically and there is still no clear sign of the Ising transition.

The above results indicate that the Ising transition may be too weak to be observed experimentally in the out-of-phase component. These results were obtained for frequencies $\leq 0.08\omega_c$, because in this range the kink in the in-phase component was most prominent. To exclude the possibility that the transition shows in a different frequency regime, we are presently studying the time correlation functions of μ and Φ. These functions cannot be measured directly but through the fluctuation-dissipation theorem, the cosine Fourier transforms are directly proportional to χ''_μ and χ''_Φ respectively. The major advantage of this method is that one simulation run gives a frequency spectrum instead of one data point at one particular frequency, as in the top graph of Fig. 1b.

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References

[1] H. Eikmans and J. E. van Himbergen, Phys. Rev. B **41**, 8927 (1990).

[2] B. Berge, H. T. Diep, A. Ghazali and P. Lallemand, Phys. Rev. B **34**, 3177 (1986).

[3] See e.g. J.M. Thijssen and H.J.F. Knops, Phys. Rev. B **42**, 2438 (1990), and references therein.

[4] H. Eikmans, J. E. van Himbergen, H. J. F. Knops, and J. M. Thijssen, Phys. Rev. B **39**, 11759 (1989).

[5] H.S.J. van der Zant and J.E. Mooij, private communications.

[6] P. Martinoli, Ph. Lerch, Ch. Leemann, and H. Beck, Jpn. J. Appl. Phys. **26**, Suppl 26-3, 1999 (1987).