

Zeitschrift: Helvetica Physica Acta
Band: 65 (1992)
Heft: 2-3

Artikel: Order and fluctuations in coupled XY planes
Autor: Zamora, M. / Chiolero, A. / Bagnoud, X.
DOI: <https://doi.org/10.5169/seals-116468>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 21.12.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Order and Fluctuations in Coupled XY Planes

M. Zamora, A. Chiolero, X. Bagnoud and D. Baeriswyl
 Institut de Physique théorique
 Université de Fribourg
 1700 Fribourg

Abstract. A three-dimensional array of classical XY spins with couplings J_{\parallel} in the planes and J_{\perp} between planes is studied both analytically and numerically. For an infinite number of planes, the magnetization is shown to be extremely sensitive to the interplane coupling. For a finite number N of planes, the magnetization vanishes, but there is a (Kosterlitz-Thouless) transition. The critical temperature increases as a function of N and rapidly approaches the bulk value.

Introduction

It is well established that the two-dimensional XY model undergoes a phase transition of a particular type, as described first by Berezinskii and by Kosterlitz and Thouless [1]. According to Mermin-Wagner theorem, below the critical temperature the order parameter remains zero, while the susceptibility is infinite. Here we consider a collection of coupled XY planes defined by the Hamiltonian

$$H = -J_{\parallel} \sum_{\ell} \sum_{\langle ij \rangle} \vec{s}_{i\ell} \cdot \vec{s}_{j\ell} - J_{\perp} \sum_{\ell i} \vec{s}_{i\ell} \cdot \vec{s}_{i, \ell+1} \quad (1)$$

where $\vec{s}_{i\ell}$ is a unit vector on site i in the ℓ 'th plane. We find that any small interplane coupling, $J_{\perp} \ll J_{\parallel}$, changes the nature of the transition [2].

Order Parameter for an Infinite Number of Planes

At low temperatures we can use the spin wave approximation for the magnetization, giving

$$m = \exp \left(-\frac{k_B T}{2J_{\parallel}} \alpha \right), \quad (2)$$

where

$$\alpha = \frac{1}{(2\pi)^3} \int_{BZ} d^3 q \frac{J_{\parallel}}{\omega(\vec{q})} \quad (3)$$

and the spin wave spectrum is

$$\omega(\vec{q}) = 4J_{\parallel} \left[\sin^2 \frac{q_x}{2} + \sin^2 \frac{q_y}{2} \right] + 4J_{\perp} \sin^2 \frac{q_z}{2}. \quad (4)$$

For $J_{\perp} \rightarrow 0$, the parameter α diverges logarithmically and the magnetization tends to zero. This is in agreement with the upper bound

$$m \leq \left(\frac{J_{\parallel}}{k_B T \alpha} \right)^{1/2}, \quad (5)$$

derived as a straightforward generalization of the Mermin - Wagner theorem [3]. Fig. 1 shows that an extremely small J_{\perp} is sufficient to efficiently suppress the fluctuations, thus stabilizing the long-range order.

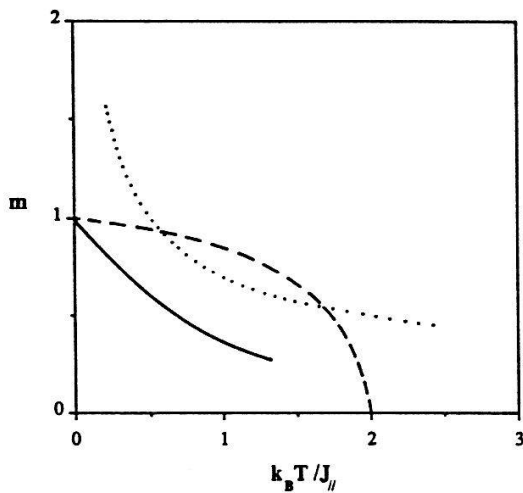


Figure 1 : Order parameter for an infinite number of planes and a ratio $J_{\perp}/J_{\parallel} = 10^{-10}$. The dashed line represents the mean-field approximation, the solid line Eq. (2) and the dotted line the r.h.s. of Eq. (5).

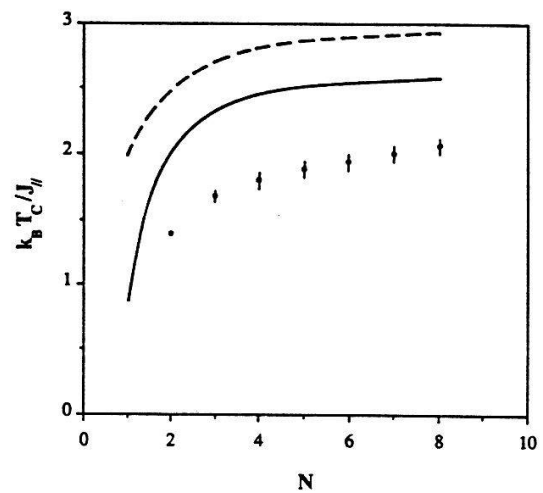


Figure 2 : Critical temperature for a finite number of planes in the isotropic case, $J_{\perp} = J_{\parallel}$. The dashed-line represents the mean field result, the full line Eq. (6) and the dots the Monte Carlo simulations.

Critical Temperature for a Finite Number of Planes

It is easy to show that for any finite number N of planes the magnetization vanishes for $T > 0$. In order to estimate the critical temperature T_c where the susceptibility diverges we treat the coupling between planes in a mean-field approximation and find the implicit relation

$$1 = 2J_{\perp} \cos\left(\frac{\pi}{N+1}\right) \chi_{2D}(T_c). \quad (6)$$

Here $\chi_{2D}(T)$ is the susceptibility of the two-dimensional XY model for which we use the high-temperature expansion of Butera et al. [4]. We have also performed Monte Carlo calculations, using a cluster algorithm and a reweighting procedure. The critical temperature was determined on the basis of the fourth cumulant of the magnetization. The results are shown in Fig. 2 for the isotropic case, $J_{\perp} = J_{\parallel}$. We see that T_c first increases rapidly as a function of N and then approaches the limiting value for the 3D XY model. For smaller values of J_{\perp} the overall increase of T_c is smaller. The comparison between the results based on Eq. (6) and the Monte Carlo data indicates that inter-plane fluctuations considerably reduce the critical temperature.

References

- [1] For a review see, e.g., P. Minnhagen, Rev. Mod. Phys. **59**, 1001 (1987).
- [2] A more detailed description of our results will be given elsewhere.
- [3] N.D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966).
- [4] P. Butera, M. Comi and G. Marchesini, Phys. Rev. B **40**, 534 (1989).