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Two-dimensional spin systems in a magnetic field close to criticality

Malte Henkel

*Département de Physique Théorique, Université de Genève
24 quai Ernest Ansermet, CH - 1211 Genève 4, Switzerland*

Abstract Numerical calculations of scaling amplitudes of correlation lengths of some 2D spin systems are reported. The results are compared with conformal theory.

Recently, it has been shown how to calculate, for two-dimensional statistical systems with a conformally invariant critical point, the universal scaling amplitudes. Consider a critical point conformally invariant Hamiltonian H_c with a relevant order parameter perturbation σ . The perturbed Hamiltonian is $H = H_c + h \int d^2x \sigma$, where h is the magnetic field conjugate to σ .

Assuming that a) the eigenstates of H can still be characterized by the eigenvalues of H_c and b) that first order terms in h are enough (since higher orders correspond to irrelevant operators at $h = 0$), Zamolodchikov [1] showed that *minimal* conformal systems can have for $h \neq 0$ non-trivial integrals of motion. This allows via a conformal bootstrap to conjecture the exact scattering matrices of the effective field theory generated by H . The S -matrices are unique up to CDD factors [2].

The 2D Ising model (at $T = T_c$) in a magnetic field is one of the systems where this theory works. This has led to ask whether this system is in fact integrable. The S -matrix contains 8 stable particles. The discrete mass spectrum is [1]

$$\frac{m_2}{m_1} = 2 \cos \frac{\pi}{5} = 1.61803\dots, \quad \frac{m_3}{m_1} = 2 \cos \frac{\pi}{30} = 1.98904\dots \quad (1)$$

Numerical checks can be done on infinitely long strips of finite width N . Particle masses m_i are related to correlation lengths ξ_i via $m_i \sim \xi_i^{-1}$. The technique used consists of diagonalising the transfer matrix (in the Hamiltonian limit) on finite lattices, followed by numerical extrapolation for $N \rightarrow \infty$. Define the scaling variable $\mu = hN^y$. Eq. (1) is valid for the limit $\mu \rightarrow \infty$ taken after the finite-size scaling limit $h \rightarrow 0, N \rightarrow \infty, \mu$ fixed. The most precise results presently available are for the Ising model [3] ($y = 15/8$)

$$\frac{m_2}{m_1} = 1.6181(5), \quad \frac{m_3}{m_1} = 1.994(5) \quad (2)$$

in good agreement with (1). One has the scaling form $m_i = h^{1/y} G_i(\mu)$ and (see [4] for details) $G_i(\mu) \simeq G_i(\infty) - \rho_i \exp(-\lambda_i \mu^{1/y})$ for $\mu \rightarrow \infty$. Calculating ρ_i gives further information on the S -matrix. The numerical results [3] are completely consistent with a *minimal* S -matrix and indicate the absence of CDD factors at least in the Ising model.

Besides the Ising model, there exist other 2D spin systems which reproduce the discrete mass spectrum (1), although *no non-trivial integrals of motion are known* and the Zamolodchikov theory is *not* applicable. The presently known examples are the tricritical Ising model ($c = 7/10, y = 77/40$) and the Ashkin-Teller model ($c = 1, y = 15/8$), both

ϵ	r_1	r_2	r_3	r_4	r_5	r_6
$-1/\sqrt{2}$	1.934(6)	2.0	-	-	-	-
-0.6	1.652(4)	2.0	-	-	-	-
-0.3	1.221(4)	1.837(6)	1.94(1)	2.0	-	-
0.0	1	1.618*	1.618	1.989*	1.989	2
0.1	1.050(3)	1.618(4)*	1.651(5)	1.99(1)*	<2.0(1)	2.0
0.25	1.125(5)	1.621(5)*	1.71(1)	1.98(2)*	<2.0(1)	2.0
0.5	1.245(5)	1.623(5)*	1.785(10)	1.99(1)*	<2.0(1)	2.0
$1/\sqrt{2}$	1.330(5)	1.615(10)*	1.79(2)	1.91(2)*	>1.95	2.0
0.875	1.370(5)	1.57(1)	1.86(1)	1.85(2)	2.00(5)	2.0
1.0	1.395(5)	1.52(5)	1.75(5)	1.8(1)	?	

Table 1: Mass ratios $r_i = m_{i+1}/m_1$ for the Ashkin-Teller model in the limit $\mu \rightarrow \infty$ for several values of ϵ covering the entire self-dual critical line. The numbers marked by * are those from the $C = +1$ sector close to (1).

perturbed by their order parameter. For the tricritical Ising model, one finds for $\mu \rightarrow \infty$ from a numerical finite-size calculation [5]

$$\frac{m_2}{m_1} = 1.62(1), \quad \frac{m_3}{m_1} = 1.98(2) \quad (3)$$

The results for the Ashkin-Teller model [6] are given in table 1 as a function of the four-spin coupling ϵ . The eigenstates of H_{AT} can be classified according to their charge conjugation $C = \pm 1$. For the $C = +1$ sector, the discrete mass spectrum in the interval $0 \leq \epsilon \leq 1/\sqrt{2}$ reproduces (1).

Summarizing, non-perturbative confirmation for the conjectured exact S -matrix was given in the $2D$ Ising model in a magnetic field. The observation that the same discrete mass spectrum is also found in models of *different* universality classes might indicate the possibility of a considerable generalization of the present understanding of slightly off-critical systems.

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