Zeitschrift: Helvetica Physica Acta
Band: 66 (1993)
Heft: 4

Artikel: Einstein's relativistic time-dilation : a critical analysis and a suggested experiment

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DOI: https://doi.org/10.5169/seals-116574

Helv Phys Acta
Vol. 66 (1993)

0018-0238/93/040346-15$1.50+0.20/0

Einstein's Relativistic Time-Dilation: A Critical Analysis and a Suggested Experiment

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(30. IX. 1992)

Abstract. This paper analyzes the experiments claimed as evidence for Einstein's relativistic time-dilation, and specifically critiques the lifetime measurement of muons in the muon $g-2$ experiments which are often thought of as the most convincing evidence for Einstein's relativistic time-dilation. The analysis, herein, indicates that so far no experimental evidence for Einstein's relativistic time-dilation could be considered completely convincing. Also, a convincing way for testing Einstein's relativistic time-dilation is suggested, avoiding logical circularity and ambiguities in experimental measurements.

The relativistic time-dilation in Einstein's special relativity [1] is the most complex and long-standing controversy in twentieth-century physics.[2] Though the controversy is not yet completely resolved theoretically, many experiments have been cited as supportive evidence for the relativistic time-dilation. Those time-dilation experiments fall into three categories: (1) measurements of transverse Doppler shift,[3–5] (2) the comparison of the reading of atomic clocks at rest and in flight,[6] (3) determinations of the decay rate of relativistic elementary particles.[7–14] Among those experiments, the lifetime measurements of muons in the $g-2$ experiments done by Bailey *et al.*[12–14] are frequently regarded as the most convincing evidence for the time-dilation. In the present paper, we, first, reexamine those time-dilation experiments to see whether or not the time-dilation is indeed confirmed experimentally. Next, we question, in detail, the reliability of Bailey *et al.*'s experiments as evidence for the time-dilation. Finally, based on Bailey *et al.*'s experiments, we suggest a much more convincing way for testing the time-dilation.

1 Reexamination of Experimental Evidence for Einstein's Relativistic Time-Dilation

1.1 Experiments of Transverse Doppler Shift

The experiments of transverse Doppler shift measures the frequency shift of light waves emitted by a light source in flight, when compared with the light frequency emitted by the same light source at rest. The observed frequency shift of the light waves emitted by a light source in straight-line flight is *interpreted* by Einstein as due to a rate slow down of clocks in flight. The experimental result is claimed to be evidence confirming the time-dilation according to Einstein's interpretation.

Some physicists do not consider Einstein's the only explanation for that transverse Doppler shift. By the principle of relativity, an equivalent view is that the observer is in flight and the light source is at rest. One must observe the same frequency shift, irrespective of viewpoint. The same experimental result seems to them to mean that Einstein's interpretation entails alternately that clocks in flight speed up rather than slow down. Consequently, the interpretation of time-dilation in special relativity for them leads to contradiction. This has been intrinsic to the unending controversy over the so-called clock (twin) paradox – pivoting on the question of how does an absolute *physical* effect of time-dilation emerge out of the relative velocities of clocks the behaviour of which should be reciprocal to each other according to the Lorentz transformation in special relativity?[2] Though the experimental results of tranverse Doppler shift are very accurate, they have not been consistently and convincingly interpreted as evidence confirming the time-dilation.

1.2 Experiments Comparing the Reading of Atomic Clocks at Rest and in Flight

The Hafele and Keating's clocks-around-the-world experiment compares the reading of atomic clocks at rest and in flight.[6] Four cesium beam atomic clocks were flown on commerical jet flights around the world twice, once eastward and once westward. The time recorded during each trip was compared with the corresponding time of a reference atomic time scale on earth. The experimental result was claimed as unambiguous evidence to resolve the clock paradox with *macroscopic* clocks. Furthermore, it was also claimed that "time dilation and the relativistic synchronization of clocks are routinely accounted for in the operation of the precision navigation and timekeeping satellites of the US Air Force's NAVSTAR global positioning system".[15]

Does the experiment of Hafele and Keating indeed resolve the clock paradox as claimed? Let us look at some criticisms of the experiment. "The data are presented only graphically in such a gross form that they cannot be examined critically. They have to be accepted, rejected, or held in question according to the personal disposition of the reader"[16] "The untreated results given in the paper indicate that the average clock lost 132 ns (nanoseconds or 10^{-9} s) for the eastward journey and gained 134 ns for the westward journey, but since the difference between individual clocks was as much as 300 ns little, if any, significance can be attached to these average values. The authors do not use all the results and apply a statistical analysis, details of which are not given, to those they do use."[17]

In addition, it was pointed out that, according to Einstein's theory of relativity, the difference of the timekeeping between polar and equatorial clocks, at the equipotential surface of the Earth, is about 104 ns/day due to the relative velocity between polar and equatorial clocks.[18] The difference is of the same order of magnitude as the observed time-dilation reported in the Hafele and Keating's experiment. In that case, the difference of the timekeeping between polar and equatorial clocks should have already been observed before the experiment done by Hafele and Keating. However, null results have been reported.[18]

In its theoretical foundation, the experiment gets complicated due to the time-dilation from Einstein's general relativity of gravitation; it no longer deals only with special relativity. [16,18] Furthermore, Cornille recently claims that he correctly *reinterprets* the Hafele and Keating's experiment for *the first time* and the acceleration has to be taken into account to resolve the clock paradox.[19] *Interpretation* of these experimental results relating to the time-dilation clearly has created a subject of controversy. Therefore, the experiment on comparing the reading of atomic clocks in flight and at rest does not convincingly confirm the time-dilation.

1.3 Experiments Determining the Decay Rate of Relativistic Elementary Particles

Experiments measuring the lifetime of elementary particles in flight are those frequently thought of as *the most convincing* evidence for the time-dilation. Among them, the recent experiments of Bailey *et al.*[12–14] are considered as the firmest evidence for the time-dilation and as definitely ending the long-standing controversy raised by the clock paradox.[20] A critical analysis of those experiments except the recent experiments of Bailey *et al.* can be found in the literature.[16,21] Thus, let us examine only the recent experiments of Bailey *et al.* in detail.

The experiments of Bailey *et al.* measure the lifetime of muons in uniform circular motion inside a storage ring. Muons in closed circular orbits mimic the journeying twin in the twin paradox who travels outward and then returns back. The journeying twin ages more slowly than the twin who stays at home, according to special relativity. Their experimental results are claimed to provide unambiguous verification of the time-dilation – the measured lifetime of the circulating muons is dilated in excellent agreement with special relativity and is unaffected by the centripetal accelerations up to about 10^{18} g (g = 9.8 m/s^2).

One may closely reexamine the experiments of Bailey *et al.*[12–14] in several aspects to see whether or not the time-dilation is indeed confirmed as claimed. It was pointed out by Waldron[22] that the measured lifetime of muons should be about 0.77 μs (μs = 10^{-6} sec) according to the lower part of Fig. 20 in the paper[14]. (Waldron might make a mistake in that he refers to Fig. 2 in the paper[12] instead of Fig. 20 in the paper[14].) Examining the lower part of Fig. 20 in the paper[14], one finds, directly from the decay of the intensity of circulating muonic bunch versus the time of flight, that the lifetime of muons is about 1.8 μs. If one takes into account the additional loss of muons due to the instability in the storage ring, the lifetime of muons might be about 2.2 μs, the lifetime of muons at rest[23]. Hence, there is no experimental evidence for the time-dilation at all, directly according to the observation of the decay of the intensity of the circulating muonic bunch versus the time of flight.

Furthermore, the period of circulating muons in the experiments of Bailey *et al.* is estimated by them as T = 147 ns, according to Fig. 2 in the paper[12] (or Fig. 18 in the paper[13]). (The radiative effect on the motion of the circulating muons due to centripetal accelerations is negligible.) The precise value of the mean period is 146.910(19) ns, which is deduced from the mean rotation frequency 6.8069(9) MHz given in the paper.[24] The mean radius of circular motion is 7.0059(7) m, as estimated from the experimental data shown in Table 4 in the paper[13] The relation between the period T and the speed of uniform circular motion v is

$$T = 2\pi r/v, \tag{1}$$

where r is the radius of circular motion. Since the mean period is 146.910 ns and the mean radius of circular motion is 7.0059 m, from Eq. 1 we obtain that the speed of muons ($\beta \equiv v/c$) is 0.99947458, where c is the speed of light. Consequently, the Lorentz factor ($\gamma \equiv (1-\beta^2)^{-1/2}$) is 30.852, and the lifetime of muons is 67.874 μs. This value is close to, but different from, the lifetime of muons 64.378(26) μs as measured by Bailey *et al.* Nonetheless, taking the uncertainty of the mean period into account, the Lorentz factor is in the range between 27.6 and 35.5. Thus, the lifetime of muons is between 60.7 and 78.1 μs. The experimental value 64.378(26) μs is not inconsistent with the values 60.7–78.1 μs as evaluated directly from the speed of muons.

However, critically reexamining the experimental results, Fig. 2 in the paper[12] (or Fig. 18 in the paper[13]), we find that the muonic bunch circulates slightly less than 27 turns during the time interval from the time 6 μs to the time 10 μs. That is the muonic bunch circulates slightly less than 27 turns in a time interval 4 μs. Hence, the mean period of circular motion of the muonic bunch should at least be 148.15 ns. With the mean radius of circular motion 7.0059 m and the period 148.15 ns, we obtain the speed of muons $\beta = 0.99110980$. This corresponds to a Lorentz factor $\gamma = 7.516$ which is substantially less than $\gamma = 29.327(4)$ as claimed in Bailey *et al.*'s experiments. *Suppose* that we accept there is a time-dilation in accordance with Einstein's special relativity, then the lifetime of muons should be equal to 16.535 μs which is about a quarter of the experimental value 64.378(26) μs. Even taking the uncertainties of the mean period and the mean radius into account, the Lorentz factor is between 6.015 and 11.458; the corresponding lifetime of muons is between 13.233 and 25.208

μs. The discrepancy found casts serious doubt upon the claimed precision in Bailey *et al.*'s experiments – claiming that the time-dilation is confirmed to high precision, a fractional error of 2×10^{-3} at 95% confidence.

The lifetime of muons found directly from either the measured period, or the decay of the intensity, of the circulating muonic bunch is inconsistent with that found by fitting the observed decay electron time spectrum with many parameters as done by Bailey *et al.*

The discrepancies found significantly reenforce criticism of the experimental evidence supposedly confirming the time-dilation. The essence of the criticism is that an unambiguous measurement of the speed of elementary particles is absent in those experiments. Thus, the speeds of particles are inferred from the relativistic expression for their energies. No convincing conclusions follow from the logically circular argument – utilizing the relativistic expression of energy in the theory to establish another relativistic expression for the time-dilation in that theory.[16,22]

In addition, a small error in the speeds of particles β would cause a large uncertainty in the Lorentz factor γ because of the rapid variation in γ, when the speeds of particles are close to the speed of light. Hence, in order to test the time-dilation within an acceptable uncertainty, one needs to measure the speed of particles with very high precision. However, in those experiments details of the claimed precision of the relativistic energies of particles are not clearly given; consequently, the values of speeds of particles inferred from the relativistic energies are very questionable. The dubiousness of the claims is shown in the above analyses where serious internal discrepancies are demonstrated in Bailey *et al.*'s experiments, which are frequently considered as the most convincing evidence for the time-dilation, owing to *lack of direct measurements of the speeds of particles.* Therefore, in those experiments the speed of particles, and thus the Lorentz factor, inferred from the relativistic energies of particles are not reliable as claimed. Furthermore, Kantor theoretically analyzed Einstein's special relativity; the time-dilation was there shown to be *physically* unobservable.[16] So far, no experimental evidence for the time-dilation could be considered completely convincing.

2 Bailey et al.'s Experiments Have Little Significance for Testing Einstein's Relativistic Time-Dilation

Some physicists may think that the discrepancy found in Bailey *et al.*'s experiments might be due to small experimental errors in measurements, for example, a minute error in measuring the period of circular motion. With a little improvement in the precision of measures, the experiments of Bailey *et al.* will again give a convincing confirmation of the time-dilation, as originally claimed. To the contrary, the following analysis will show that the experiments of Bailey *et al.* can not provide a convincing test for the time-dilation.

According to the Lorentz-covariant electromagnetic force law in special relativity, for a particle of mass m and charge q in uniform circular motion with speed v in a uniform magnetic field B, we have

$$r = (mv/qB)\,(1 - (v/c)^2)^{-1/2}, \tag{2}$$

where r is the radius of circular motion. From Eq. 1 and Eq. 2, we obtain the predictions of v/c versus magnetic field and period versus magnetic field shown in Fig. 1 and Fig. 2, respectively. Also, the prediction of the Lorentz factor versus period is shown in Fig. 3.

Here, we use the mass of muon $m = 1.8835326 \times 10^{-28}$ kg, the charge $q = 1.60217738 \times 10^{-19}$ C and the speed of light $c = 2.99792458 \times 10^{8}$ m/sec,[25] as well as the mean radius of circular motion $r = 7.0059$ m. From these values, one obtains for the period of circular motion 146.919–146.918 ns for the magnetic field varying in the range 1.47–1.48 tesla. (The magnetic field set up in the experiments of Bailey *et al.* is about 1.472 tesla.[12,13]) In addition, when the period is 146.910 ns, the magnetic field should be at 1.55 tesla.

The discrepancy in the mean period between the experimental observation and the theoretical prediction is about 1.23 ns (148.15 − 146.92 ns). Is the found discrepancy due to the experimental errors in measuring the mean radius or the magnetic field? From Eqs. 1 and 2, we have as the uncertainty of the period ΔT related to the uncertainties of the mean radius and the magnetic field, Δr and ΔB respectively,

$$\frac{\Delta T}{T} \sim [\, (\frac{4\pi^2 r^2}{c^2 T^2} \frac{\Delta r}{r})^2 + (\frac{4\pi^2 m^2}{B^2 q^2 T^2} \frac{\Delta B}{B})^2 \,]^{1/2}. \tag{3}$$

Suppose that the uncertainties in the mean radius and the magnetic field are allowed to be 2 mm and 100 gauss, respectively; these assumed uncertainties are larger than the corresponding experimental errors mentioned in Bailey *et al.*'s experiments.[12–14] (The uncertainty of the magnetic field in the experiments is claimed as only a few micro-tesla.) Corresponding to the assumed uncertainties, from Eq. 3 one finds that the uncertainty in the mean period is only 0.042 ns, at a magnetic field of 1.472 tesla and a mean radius of 7.0059 m as in Bailey *et al.*'s experiments. Thus, the found discrepancy in the period 1.23 ns can not be attributed to the experimental errors in the mean radius and the magnetic field.

By the definition of the Lorentz factor and Eq. 1, the relationship of the uncertainties in the Lorentz factor, the mean radius and the mean period is given as

$$\frac{\Delta\gamma}{\gamma} \sim (\gamma^2 - 1)\frac{\Delta\beta}{\beta} \sim (\gamma^2 - 1)\, [\, (\frac{\Delta T}{T})^2 + (\frac{\Delta r}{r})^2 \,]^{1/2}. \tag{4}$$

From Eq. 4, one sees that errors in the measurements of the mean period and the mean radius cause uncertainties in the speed of muons and thus in the Lorentz factor. In addition, the Lorentz factor is very sensitive to the change of the speed of muons, especially when the speed is close to the speed of light. Thus a small error in the speed could cause a large error in the Lorentz factor. This can be seen in the theoretical predictions, Fig. 1 and Fig. 2, as the magnetic field varies from 0.5 tesla to 2.5 tesla, the speeds of muons are very close to the speed of light and the periods of circular motion vary between 146.8 ns and 147.6 ns. The variation of the period of circular motion is only about 0.8 ns, for the magnetic field varying between 0.5 and 2.5 tesla. To be more specific, when the period is 147 ns (corresponding to the speed $\beta = 0.99886265$), the Lorentz factor γ is 20.973; when the period is 146.91 ns (corresponding to the speed $\beta = 0.99947458$), the Lorentz factor γ is 30.852. The difference in the period is only 0.09 ns, but the corresponding difference in the Lorentz factor is about 9.9.

The precision of the clocks utilized in the experiments is claimed as precise as ≈ 20 ps (10^{-12} sec).[14,26] The statistical error on the mean radius of the circulating muons mentioned in the experiment is typically 0.1–0.2 mm.[13,26] The experimental data in Table 4 of the paper[8] suggest that the overall uncertainty of the mean radius is about 0.7 mm. Suppose that the uncertainties in measuring the mean period and the mean radius are 0.02

ns and 0.5 mm, respectively. Then, corresponding to the assumed precisions, the fractional uncertainty of the Lorentz factor $\frac{\Delta\gamma}{\gamma}$ is about 13%. In this case, the experiments of Bailey *et al.* would have little, if any, significance for testing the time-dilation.

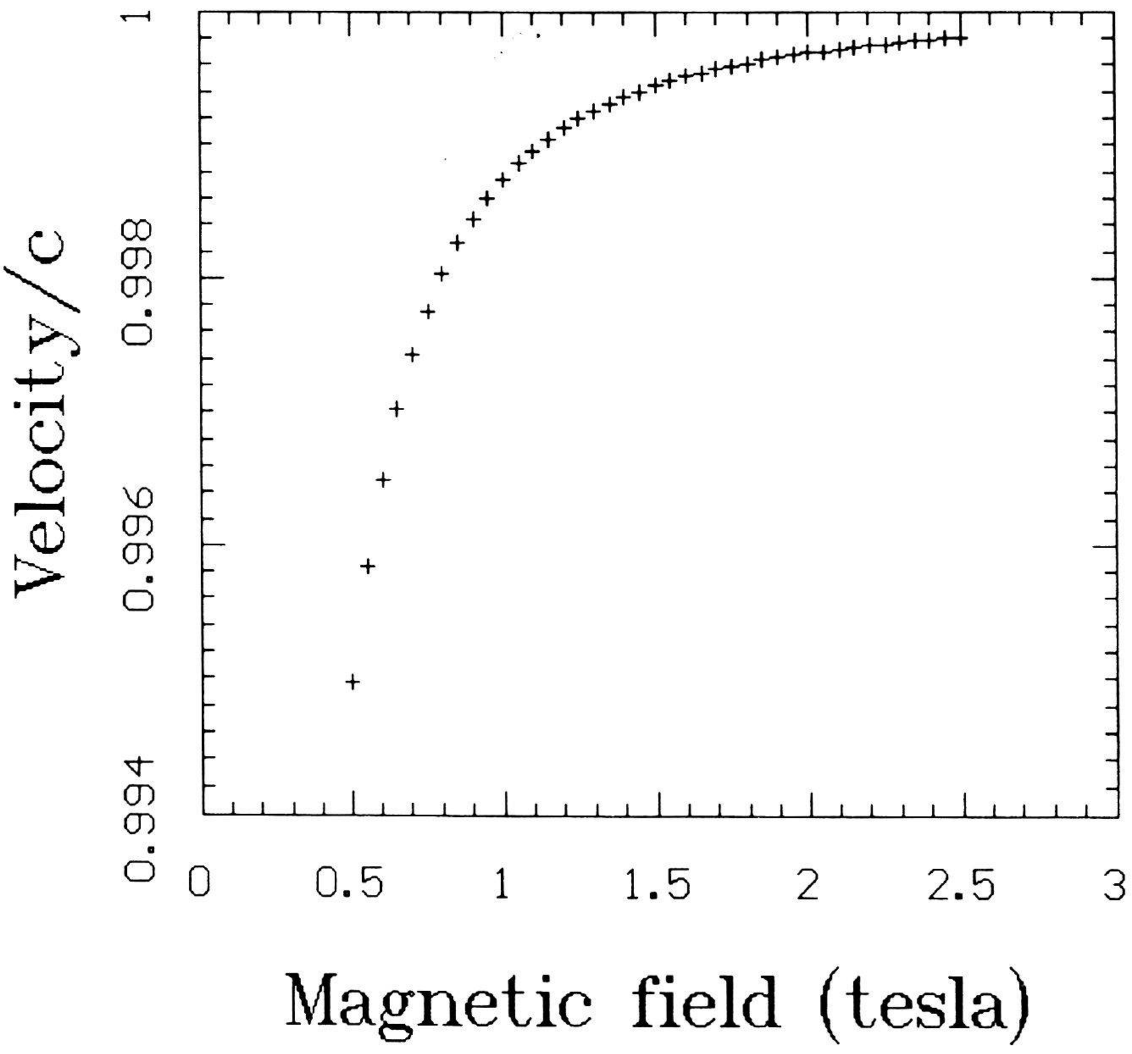

Figure 1: Theoretical prediction of v/c vs. magnetic field for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 7.0059 m.

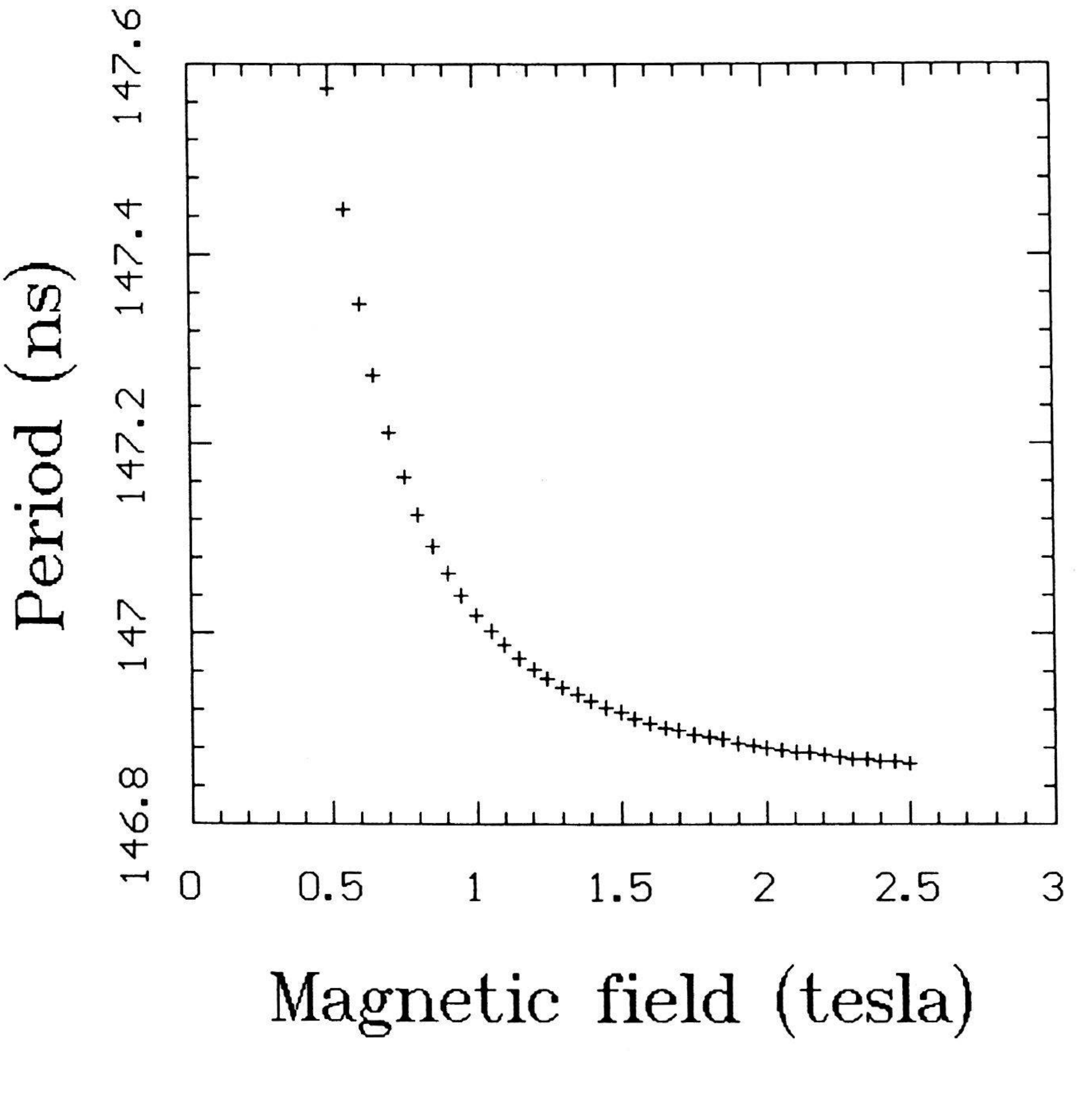

Figure 2: Theoretical prediction of period vs. magnetic field for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 7.0059 m.

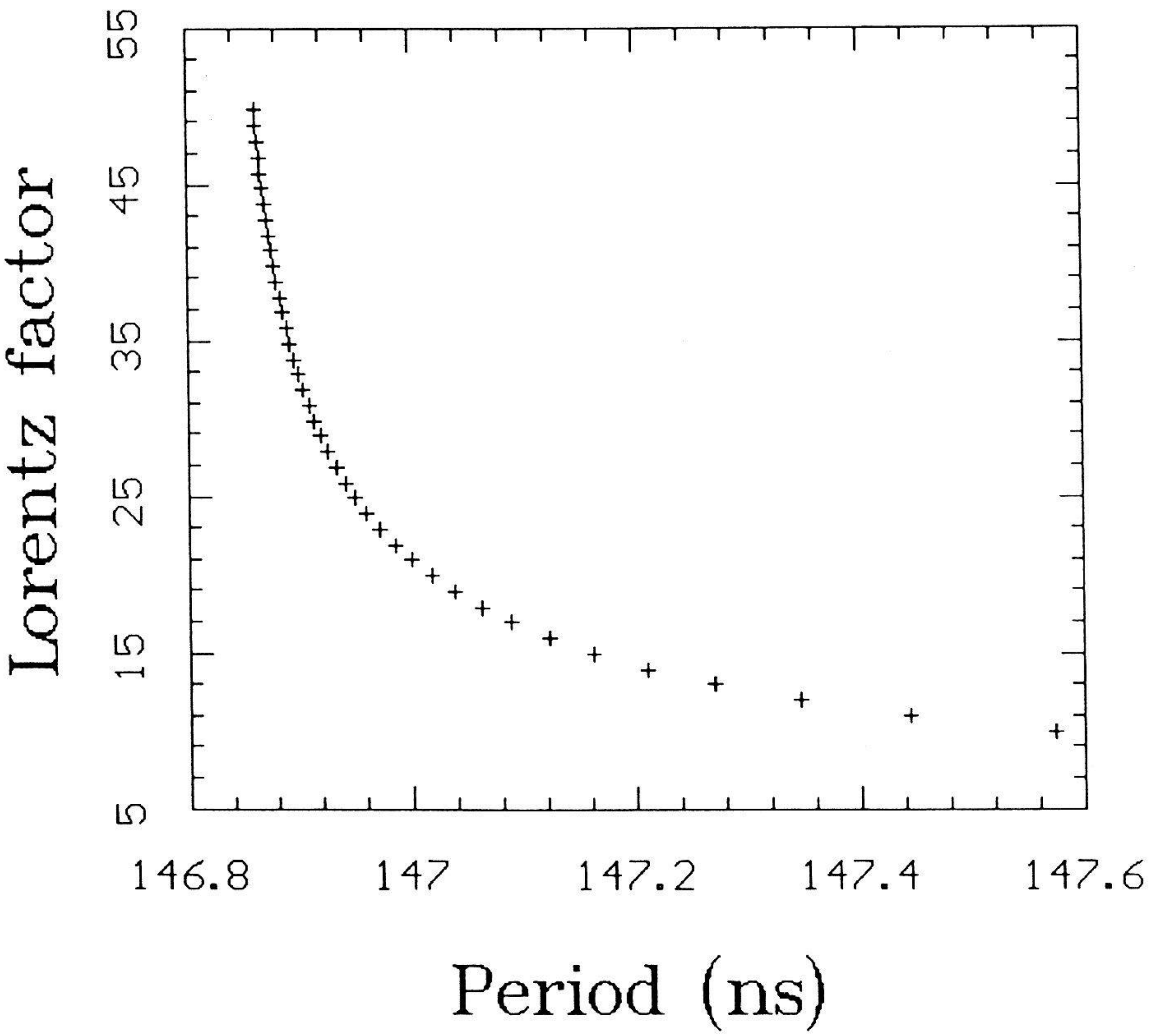

Figure 3: Theoretical prediction of Lorentz factor vs. period for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 7.0059 m.

3 A Convincing Way for Testing Einstein's Relativistic Time-Dilation

It should be stressed that the serious discrepancies found in the experiments claimed as evidence for the time-dilation are due to a lack of direct measurement of the speed of particles. To overcome this stressed weakness in experimental evidence for the time-dilation, we suggest performing an experiment, similar to Bailey *et al.*'s experiments, on muons with speeds *lower* than that in Bailey *et al.*'s experiments. In the suggested experiment, the Lorentz factor should be evaluated from the speed of particles which should be directly determined from experimental measurements. In order to perform the experiment on muons with lower speeds, we suggest constructing a circular storage ring with a smaller mean radius, for example, 1 meter. The predictions of v/c versus magnetic field, period versus magnetic field and Lorentz factor versus period are shown in Fig. 4, Fig. 5 and Fig. 6, respectively. Here, we use 1 m as the mean radius of circular motion. From Fig. 4 and Fig. 5 we see that the speed β varies from ≈ 0.81 to ≈ 0.99 and the period varies from ≈ 21 ns to ≈ 26 ns, for the magnetic field varying from 0.5 tesla to 2.5 tesla. Since the speed of muons is not as close to the speed of light as that in Bailey *et al.*'s experiments, the Lorentz factor is not so sensitive to the change of the speed of muons. Also, from Fig. 6 we see that the Lorentz factor is much less sensitive to the change of period than that for muons with near light-speed as shown in Fig. 3. More specifically as shown in Fig. 6, when the period is 24.2 ns, the Lorentz factor is ≈ 2; when the period is 22.2 ns, the Lorentz factor is ≈ 3; when the period is 21.4 ns, the Lorentz factor is ≈ 5. The difference in the period is 2.8 ns, but the corresponding difference in the Lorentz factor is only about 3.

Let us assume that the measurements in the mean period and the mean radius could be achieved to such high accuracy as 0.01 ns and 0.5 mm respectively, which might be on the limits of present technology. Then for a mean radius of 1 m, one has, for example, $\frac{\Delta\gamma}{\gamma} \approx 1.2\%$ ($\gamma = 4.289$) at a magnetic field of 1.47 tesla, and $\frac{\Delta\gamma}{\gamma} \approx 0.6\%$ ($\gamma = 3.008$) at a magnetic field of 1 tesla. The uncertainty in the Lorentz factor in the suggested experiment is much lower than that of Bailey *et al.*'s experiments. Therefore, within present technology performing the experiment on muons with lower speeds should be more reliable than that on muons with speeds extremely near the speed of light.

Here, we wish to emphasize that a convincing way for testing the time-dilation should be performed as follows.

1. Simultaneously measure the decay electron time spectrum as that shown in the upper part of Fig. 20 in the paper[14], and the muonic decay intensity time spectrum as that shown in Fig. 2 in the paper[12] (or Fig. 18 in the paper[13]).

2. Determine the lifetime of muons directly from both the muonic decay intensity time spectrum and the decay electron time spectrum, and then check whether these two measured values of the muon lifetime are consistent.

3. Directly from the muonic decay intensity time spectrum, determine the mean period of circular motion. Then, calculate the speed of muons by the *time-of-flight method* with the measured values of period and radius of circular motion.

Finally, from the corresponding values of lifetime and speed of the circulating muonic bunch, one can systematically test whether there is a time-dilation in accordance with the theory of special relativity. Since the speed and lifetime of muons are determined directly from experimental measurements, the suggested experiment involves neither ambiguities in the physical meaning of experimental measurements nor the presumption of special relativity. If the suggested experiment can be done within present technology, then it will be more convincing than Bailey *et al.*'s experiments for testing the time-dilation.

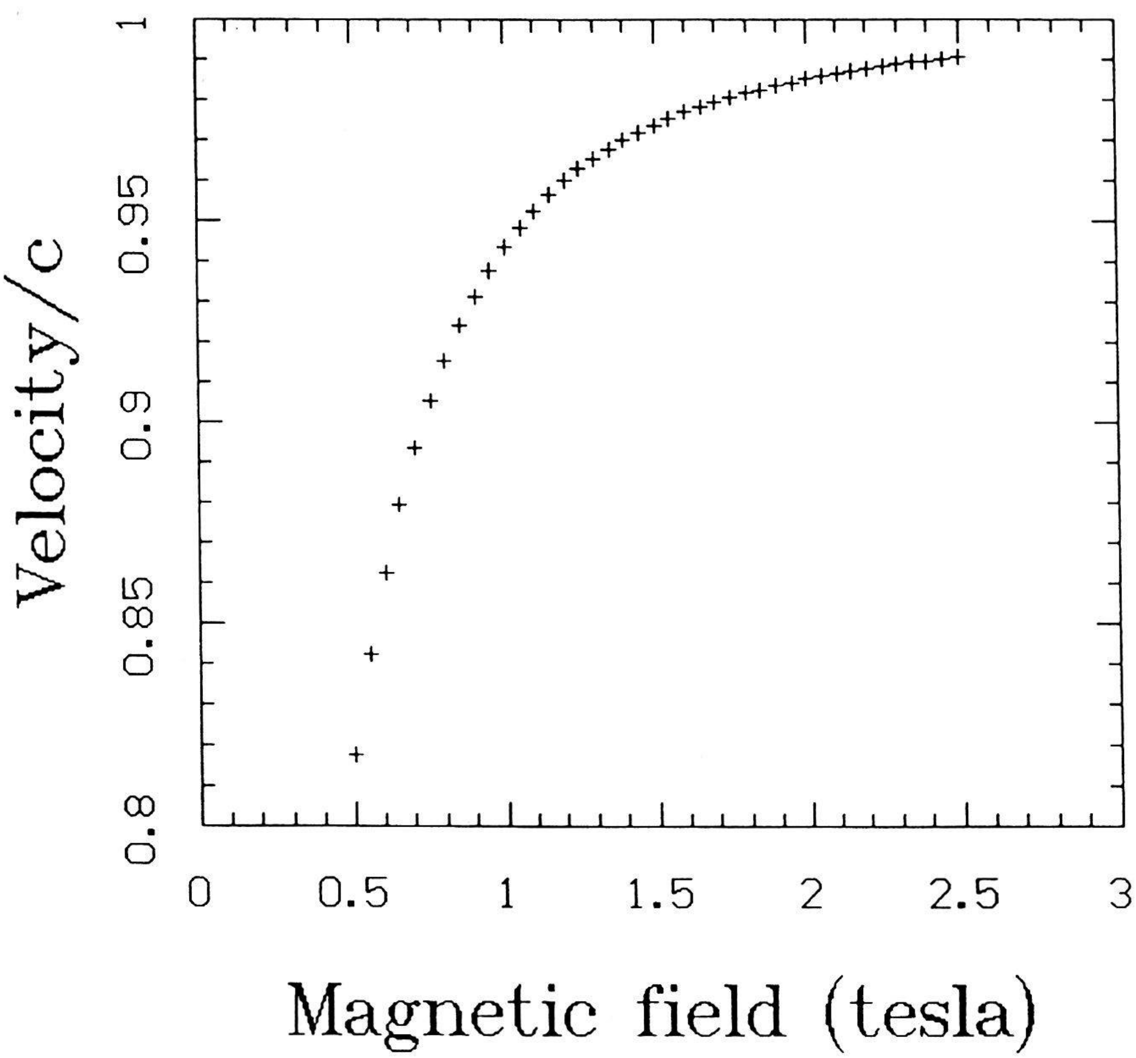

Figure 4: Theoretical prediction of v/c vs. magnetic field for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 1 m.

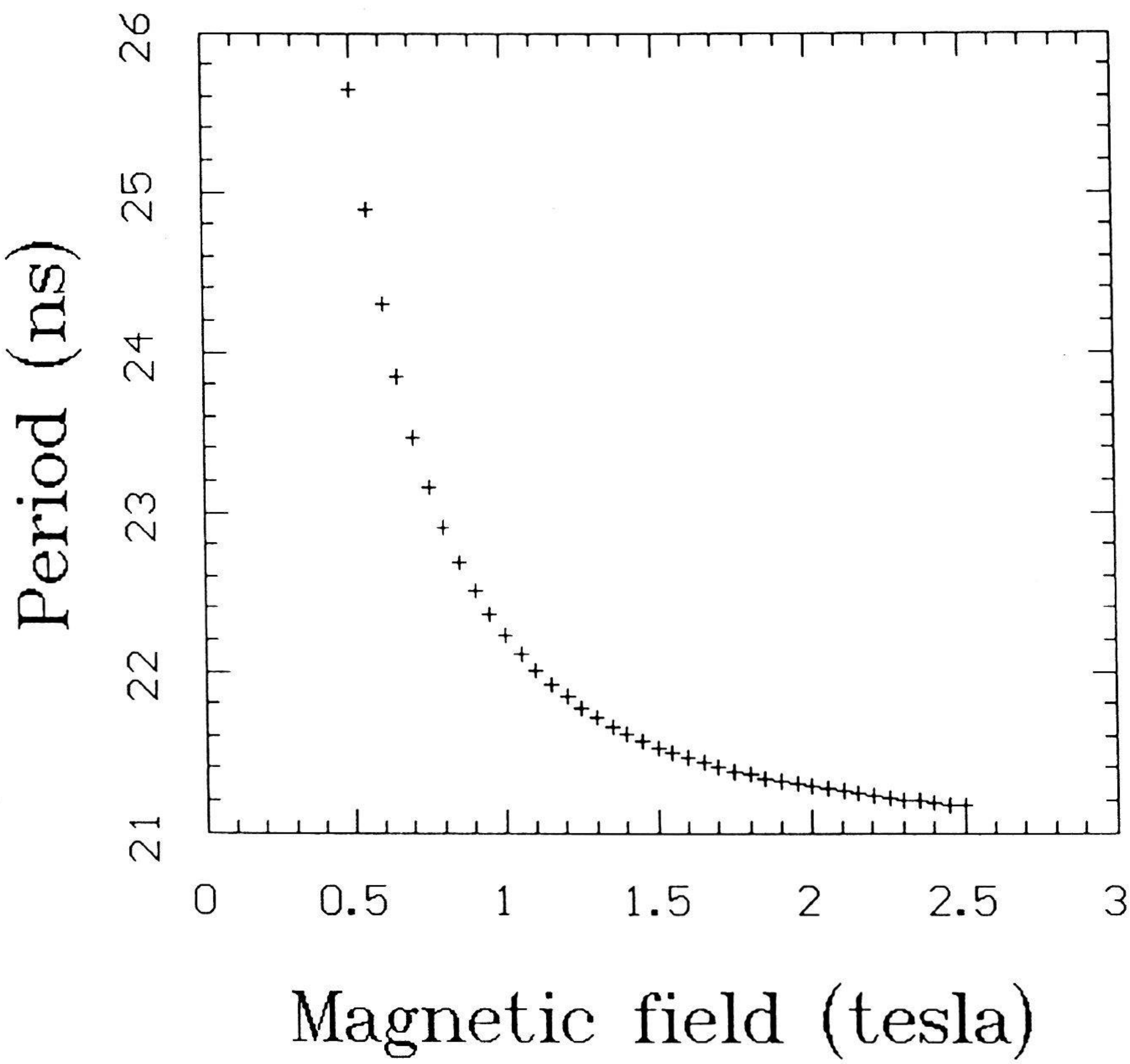

Figure 5: Theoretical prediction of period vs. magnetic field for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 1 m.

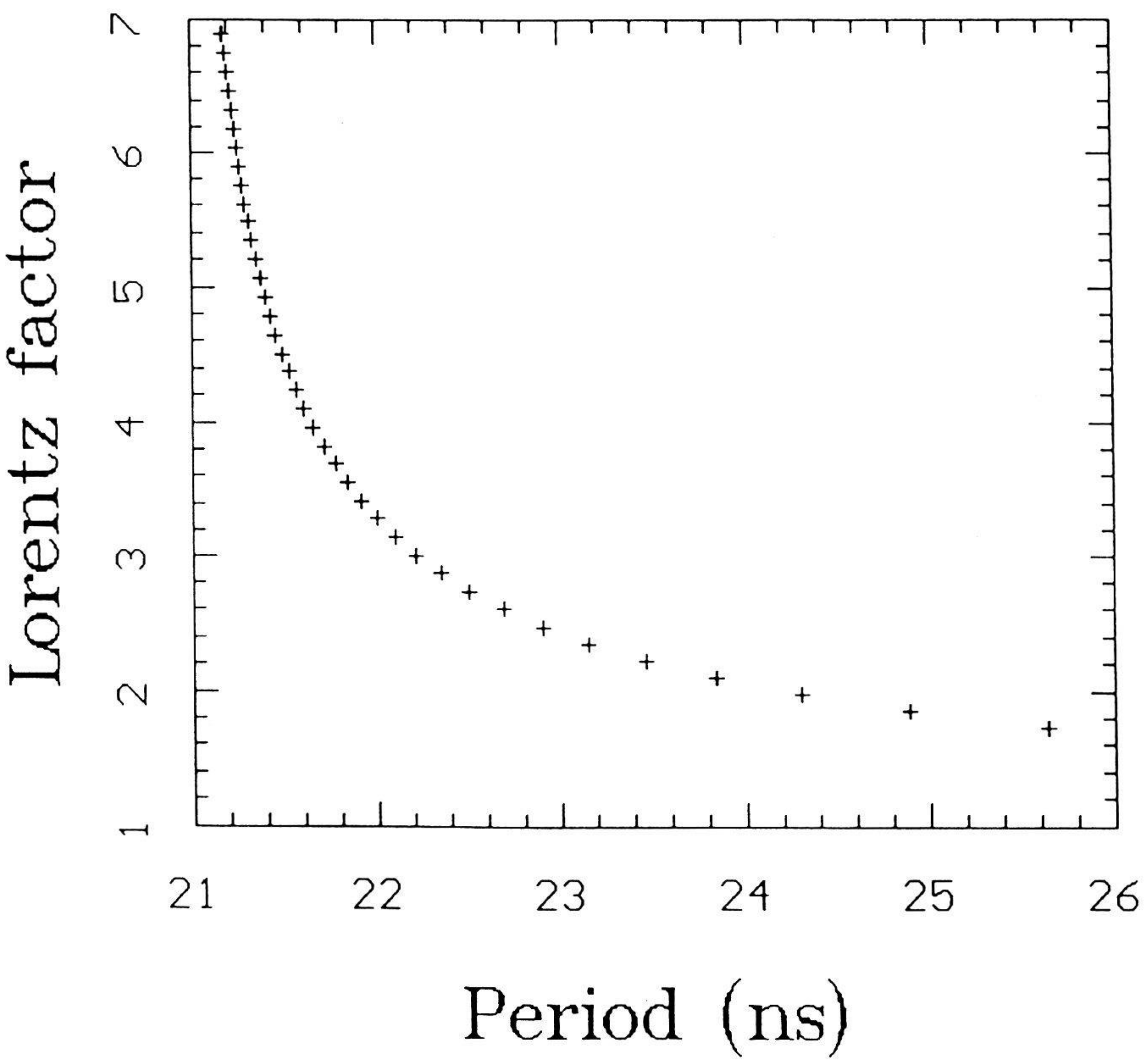

Figure 6: Theoretical prediction of Lorentz factor vs. period for muons in uniform circular motion in a uniform magnetic field, according to the Lorentz force law. The radius of circular motion is 1 m.

4 Conclusions

Serious discrepancies are pointed out in the lifetime measurements of muons by Bailey *et al.* which are acclaimed as the most convincing evidence for the time-dilation. In addition, the detailed analysis of Bailey *et al.*'s experiments indicates that the experiments have little significance for testing the time-dilation. So far, experimental evidence for the time-dilation is inconclusive. A clean experiment to systematically test the time-dilation is suggested. We particularly emphasize a convincing way to conduct the suggested experiment: the lifetime of muons must be measured directly from the experimental data showing the decay intensity of the muonic bunch versus the time of flight, and the speed of muons must be determined from *the same* set of experimental data by the time-of-flight method. The suggested experiment is based on direct measurements, and involves neither logically circular arguments nor ambiguities of interpretation. With modern technology, this proposal experiment might provide a convincing test for the time-dilation.

Acknowledgement

The author gratefully thanks Dr. C.M.L. Leonard for valuable comments during the preparation of this paper.

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