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# Comment on "Normalization of scattering states, scattering phase shifts, and Levinson theorem" by N. Poliatzky [Helv. Phys. Acta 66, 241–263 (1993)]

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In the recent paper [1] Poliatzky claims to have derived a Levinson theorem for the Schrödinger equation for a central potential that differs from the well known result in allowing for the possiblity that

$$\eta_0(0) = \left(n_0 + \frac{1}{4}\right)\pi, \tag{0.1}$$

where $\eta_0(0)$ is the s-wave phase shift at zero energy and $n_0$ the number of s-wave bound states.

The author's method of derivation relies on manipulations of the Dirac distribution that are of very dubious validity. He arrives at a modification of the usual quasi-orthogonality of the scattering wave functions in which the Dirac distribution is supplemented by a bounded function that differs from zero at only one point. In any context in which the Dirac distribution is meaningful, such a function is of course equal to zero, and its addition is of no consequence.

Quite apart from the dubious methods of its derivation, there is the more important question of the validity of the author's result of a modified Levinson theorem. It is clear from the usual derivation by means of a contour integral (see, for example [2], page 356) that (1) implies that the Jost function for $l = 0$ at the origin must be proportional to $k^{1/2}$. Such a behavior is known [3] to be ruled out if $\int_0^\infty dr(1+r)|V| < \infty$. Therefore, if the potential decays as fast as $r^{-2-\epsilon}$ with some $\epsilon > 0$, (1) cannot happen. The Levinson theorem for potentials that decay more slowly than that was discussed in [4], and the results generally disagree with (1). However, the author claims that (1) may hold only if there is a "zero-energy resonance," and he also claims (without proof) on lines 5-6 of page 254 that "If the potential does not vanish faster than $1/r^2$ at infinity, then finite, zero-energy, solutions do not exist." The implication is that (1) is possible for potentials that vanish faster than $1/r^2$. This is incorrect.

The author's derivation of the high-energy behavior of the phase shift for potentials that decay like $r^{-2}$ or more slowly (but faster than $r^{-1}$) is also subject to doubt. His change of variables leads, without comment, to a highly singular equation on a complex contour, and it is not clear whether his manipulations are valid.

I will not comment on the Dirac case, but a relevant comparison here is reference [5]. Finally, contrary to the author, I cannot take credit for the version (5) of the Levinson theorem. The extra $\frac{1}{2}\pi$ already appeared in Levinson's original paper [6].

## References

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