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Some Remarks on the Algebra of Supercharges in D=(1+2)

By L. P. Colatto[1]

DCP, Centro Brasileiro de Pesquisas Físicas
CEP 22290-180 Rio de Janeiro, RJ, Brazil

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Abstract. One shows that 3-dimensional N=1-supersymmetric models with non-trivial topological field configurations develop a central charge in the supersymmetry algebra. This result is also discussed in the presence of a Chern-Simons term, keeping supersymmetry non-extended.

1 Introduction

Ordinary and supersymmetric Abelian gauge models in 3D space-times have been fairly-well investigated over the past years [1]. Besides their relevance in connection with the possibility of getting non-perturbative results more easily, the ultraviolet finiteness of Chern-Simons (including the case of gravity) theories is a remarkable feature of field theories defined in D=(1+2) [2]. Also, Abelian Chern-Simons models with matter couplings seem to be the right way to tackle exciting topics in Condensed Matter Physics, such as high-T_c superconductivity and Fractional Quantum Hall Effect [3]. Our purpose in this note is to assess a typical three-dimensional gauge model with N=1-supersymmetry from the point-of-view of the algebra of the supersymmetry generators. We actually wish to present here a few remarks on the connection between topologically non-trivial solutions, the Chern-Simons term, and the presence of a central charge operator in the algebra of *simple* supersymmetry.

[1]e-mail: colatto@cbpfsu1.cat.cbpf.br

The (N=1)-super-Poincaré algebra in $(1+2)$ dimensions is generated by a real two-component spinorial charge, Q_a, whose the operatorial relations are given by $\{Q_a\,,\,Q_b\} = 2\,i\,P_{ab}$ and $[Q_a\,,\,P_{ab}] = 0$, P_{ab} being the translation generator [4]. This super-Poincaré algebra has been generalized in [5] for extended supersymmetries. But, in fact, to recognize if a quantum field theory is consistent with supersymmetry, we need to obtain the Noether's (super)charges for the specific model, and the local features of a system are presented by the current algebra, that depends, as we will see, on the details of the model [6]. It is worthwhile to mention that this is one way to detect the symmetry at the quantum level; another approach would be through the analysis of the Ward identities for the symmetry under consideration. With this point-of-view, we shall analyse how the various terms (consistent with the symmetries of the (1+2)-dimensional supersymmetric model) contribute to the equal-time commutators. The motivation to use the canonical formulation in our analysis is that we actually wish to identify a central charge in the algebra and, in case it appears, it can be read off from the RHS of the anticommutators between the supersymmetry charges.

The models we shall contemplate here include an N=1-supersymmetric self-interacting matter model, its coupling to gauge fields and an N=1-supersymmetric Abelian Chern-Simons model. The main result we find is that a central charge appears, even if the supersymmetry is simple, provided that topologically non-trivial solutions are present in the model.

The explicit representation we adopt for the symmetric $\tilde{\gamma}$-matrices ($\tilde{\gamma} \equiv C\,\gamma$) reads:

$$(\tilde{\gamma}^0)_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \;,\; (\tilde{\gamma}^1)_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \;,\; (\tilde{\gamma}^2)_{ab} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

where C is the charge conjugation:

$$C = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} . \tag{1.1}$$

As a notational convention, we shall denote spinor indices by early latin characters (a,b,...).

2 The Self-Interacting Matter Model

Matter fields are assembled in scalar superfields, whose representation in terms of a θ-expansion (θ is a two-component Grassmann-valued Majorana spinor) is given by $\Phi(x,\theta) = \phi(x) + \theta^a\,\psi_a(x) - \theta^2\,F(x)$, where $\phi(x)$ is a physical scalar, $\psi_a(x)$ is a physical fermion and $F(x)$ is an auxiliary field. Now, it is possible to write an action that is invariant with respect to the symmetries (supersymmetry, Lorentz) and that is power-counting renormalizable:

$$S_{\rm matter} \;=\; \int d^3x\,d^2\theta \left\{ -\frac{1}{2}\,(D_a\,\Phi)^2 + \frac{1}{2}\,m\,\Phi^2 + \frac{\lambda}{8}\,\Phi^4 \right\} ,$$

$$D_a \equiv \left(\frac{\partial}{\partial\theta^a} - i\,\theta^b\,\frac{\partial}{\partial x^{ab}}\right)\Phi \equiv (\partial_a - i\,\theta^b\,\partial_{ab})\,\Phi\,. \tag{2.1}$$

The supercharge is obtained via Noether theorem and reads as below:

$$\begin{aligned} Q_c &= \int d^2\vec{x} J^0{}_c \\ &= \int d^2\vec{x}\left\{-i\,\psi^a\,(\tilde{\gamma}^0)_{ac}\left(m\,\phi + \frac{\lambda}{2}\phi^3\right) + \frac{1}{2}\psi_c\,\partial^0\,\phi + \frac{1}{2}\,\phi\,\partial^0\,\psi_c + \right. \\ &\quad \left. -\frac{i}{2}\,\varepsilon^{0\,\nu\,\rho}\,\psi^a\,(\tilde{\gamma}_\rho)_{ac}\,\partial_\nu\,\phi + \frac{i}{2}\,\varepsilon^{0\,\nu\,\rho}\,\phi\,\partial_\nu\,\psi^a\,(\tilde{\gamma}_\rho)_{ac}\right\}\,. \end{aligned} \tag{2.2}$$

By using the graded canonical commutation relations for ϕ and ψ_a, the anticommutator defining the super-Poincaré algebra turns out to be:

$$\{Q_a\,,\,Q_b\} = -2\,i\,P^\mu\,(\tilde{\gamma}_\mu)_{ab} + i\,\epsilon^{ij}\int d^2\vec{x}\,\partial_i\left[(\tilde{\gamma}_j)_{ab}\left(m\,\phi^2 + \frac{\lambda}{2}\phi^4\right)\right]\,, \tag{2.3}$$

where P^μ is obtained as the $0\,\mu$ component of the "improved" energy-momentum tensor. Observe that the mass and self-interaction terms both contribute to a central charge in the supercharge algebra. The case we study here is just the 3-dimensional counterpart of the model considered by Olive and Witten in the paper of ref. [7], where the topological origin for the central charge is first pointed out. In our case, proceeding along the same lines as done in ref. [7], it is shown that the second term in the RHS of eq. (2.3) is non-trivial by virtue of the contributions coming from soliton-like configurations associated to the ϕ-field. However, it should be stressed that these static configurations are not the straightforward generalization of the kinks to two space dimensions, for they have an energy that is divergent. They simply signal the presence of field configurations with non-trivial topology and only upon the introduction of gauge fields, as we shall proceed to in the next section, they will lead to static configurations of *finite energy*: they are the so–called solitons with a magnetic flux, or vortices [8].

3 Matter–Gauge Background Coupling

The next step of our discussion consists in complexifying the matter model of the previous section and then performing the gauging of the $U(1)$-symmetry it possesses [4]. The degrees of freedom in the gauge sector are accommodated in a Majorana spinor superfield, $\Gamma_a(x;\theta)$, whose θ-expansion reads [4]:

$$\Gamma_a = \chi_a + \theta^b(C_{ab}B + iV_{ab}) + \theta^2(2\lambda_a - i\partial_a{}^b\chi_b)\,, \tag{3.1}$$

where V_{ab} is the usual $U(1)$-gauge field, λ_a is its physical supersymmetric partner (the photino), whereas χ_a and B are non-physical components; C_{ab} denotes the elements of the charge conjugation matrix. The action that accomplishes the minimal coupling between the matter and gauge sectors is given by [4]

$$S_{\text{matter-gauge}} = \int d^3x\, d^2\theta \Big\{ -\frac{1}{2} (\nabla^a \bar{\Phi})(\nabla_a \Phi) \Big\} \,, \tag{3.2}$$

whith $\nabla_a \Phi \equiv (D_a - i\Gamma_a)\,\Phi = (\partial_a - i\theta^b \partial_{ba} - i\Gamma_a)\,\Phi$. The Noether supersymmetry charge turns out to be

$$\begin{aligned} Q_c &= \int d^2\vec{x} J^0{}_c = \\ &= \frac{1}{4} \int d^2\vec{x} \Big\{ i\,\varepsilon^{0ij} \, [\bar{\psi}^a \, (\tilde{\gamma}_j)_{ac} \nabla_i \, \phi + \psi^a \, (\tilde{\gamma}_j)_{ac} \nabla_i \, \bar{\psi}] - (\psi_c \nabla^0 \, \bar{\psi} + \bar{\psi}_c \nabla^0 \, \phi) + \\ &- \, (\bar{\phi} \, \nabla^0 \, \psi_c \, + \, \phi \, \nabla^0 \, \bar{\psi}_c) \, + \, i\,\varepsilon^{0ij} \, (\bar{\phi} \, \nabla_i \, \psi^a \, + \, \phi \, \nabla_i \, \bar{\psi}^a)(\tilde{\gamma}_j)_{ac} \Big\} \,; \end{aligned} \tag{3.3}$$

∇^0 and ∇^i stand for the gauge-covariant derivatives with space-time indices. Also, we should mention that our component fields are defined through the action of the (spinorial) gauge-covariant derivatives on the superfield Φ, according to:

$$\begin{aligned} \Phi|_{\theta=0} &= \phi \,, \\ \nabla_a \, \Phi|_{\theta=0} &= \psi_a \,, \\ \nabla^2 \, \Phi|_{\theta=0} &= F \,. \end{aligned} \tag{3.4}$$

After a lengthy computation and the use of well-known algebraic relations among the $\tilde{\gamma}$-matrices, we obtain that

$$\{Q_a \,,\, Q_b\} = -2\,i\,P^\mu (\tilde{\gamma}_\mu)_{ab} \,, \tag{3.5}$$

where te momentum operator P^μ appearing in the RHS includes now contributions from the gauge field minimally coupled to matter through (3.2). Nevertheless, no term in the form of a central charge arises from the action (3.2); this means that the central charge operator of eq. (2.3) is not modified by the introduction of the $U(1)$ gauge superfield. The role of the latter is to stabilize the topological configurations associated to the action (2.1) in the form of vortex-like solitons, as already known from the works quoted in ref. [8].

4 The N=1 Abelian Chern-Simons Term

The gauge sector discussed in the previous section was treated as a background; since no Maxwell-like dynamical term was added to the action (3.2) that gives propagation to the physical fields V_{ab} and λ_a. We have now in mind to introduce a Chern-Simons (CS) term for the latter and we shall compute how it contributes to the algebra of eq. (3.5). We begin with the gauge-invariant CS term written up in superspace [4]:

$$S_{CS} = \frac{M}{g^2} \int d^3x \, d^2\theta \, \Gamma^a \, W_a \,, \tag{4.1}$$

where M is a mass parameter, g is the gauge coupling constant and the field-strength superfield W_a is defined as $W_a = \frac{1}{2} D^b D_a \Gamma_b$ [4]. The CS term above contributes the piece

$$(Q_{CS})_c = \int d^2\vec{x} (J_{CS})^0_c = \int d^2\vec{x} \Big\{ \frac{i}{2} V^0 \left(\psi_c \bar{\phi} - \bar{\psi}_c \phi \right) \Big\} \,, \tag{4.2}$$

to be added to the RHS of the supersymmetry charge of eq. (3.3). The reader should be warned for the fact that the contribution given in eq. (4.2) is true once one has chosen to work in the Wess-Zumino gauge. Moreover, also restricting the supercharge of eq. (3.3) to the Wess-Zumino gauge, one computes the anticommutator (3.5) with the CS piece and verifies that no new contribution appears that modifies the central charge.

5 Conclusions

The work of Haag, Lopusanski and Sohnius [5] yields a theorem stating that a central charge in a supersymmetric model in four dimensions can only appear whenever supersymmetry is extended. In two dimensions, Olive and Witten [7] showed how configurations with non-trivial topology (namely, kinks) could be the source for a central charge operator in the framework of a simple supersymmetry. In this letter, we presented some results on the algebra of supercharges for a self-interacting matter model and an Abelian gauge theory in $D=(1+2)$. Our results indicate that the conclusion drawn from the two-dimensional case persists in three dimensions, i. e., 3-dimensional models with topologically non-trivial field configurations develop a central charge in the algebra of *simple supersymmetry*. The central charge appears indeed as a space integral that probes the behaviour of the fields at infinity.

This result differs from the one presented in the paper of ref. [9], where the authors consider an Abelian ($N=2$)-supersymmetric model with a ϕ^6-potential and a Chern-Simons term for the gauge potential. They conclude that the non-trivial topological contribution to the central charge is nothing but the magnetic flux associated to the gauge field.

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References

[1] A. Linde, *Rep. Progr. Phys.* **42** (1979) 389;
S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* **48** (1982) 975;
S. Deser, *Three Topics in Three Dimensions*, in the proceedings of the Trieste Conference on Supermmenbranes and Physics in 2+1 Dimensions, eds. M.J. Duff *et al.*, World Scientific (Singapore, July 1989);

[2] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B346** (1990) 313;
A. Blasi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B356** (1991) 154;

C. Lucchesi and O. Piguet, *Nucl.Phys.* **B381** (1992) 281;

[3] J. Schonfeld, *Nucl. Phys.* **B185** *(1981) 157*;
Fractional Statistics and Anyon Superconductivity, ed. by Franck Wilczek, World Scientific (Singapore), 1990;

[4] S. J. Gates, M. T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin/Cummings Publishing Company (1983);

[5] S. Coleman and J. Mandula, *Phys. Rev.* **159** (1967) 1251;
R. Haag, J. T. Lopuszanski and M. Sohnius, *Nuc. Phys.* **B88** (1975) 257;

[6] R. Jackiw, *Field Theoretic Investigation in Current Algebra, Current Algebra and Anomalies*, S. B. Treiman, R. Jackiw, B Zumino and E. Witten (eds.), World Scientific Publishing Co. (1985);

[7] E. Witten and D. Olive, *Phys. Lett.* **B78** (1978) 97;
P. Di Vecchia and S. Ferrara, *Nuc. Phys.* **B130** (1977) 93;
A. D'Adda and P. Di Vecchia, *Phys. Lett.* **B73** (1978) 162;

[8] H. B. Nielsen and P. Olesen, *Nucl. Phys.* **B61** (1973) 45;
see also L. H. Ryder, *Quantum Field Theory* - Chapter 9, Cambridge University Press, 1985;

[9] C. Lee, K. Lee and E. Weinberg, *Phys. Lett.* **B243** (1990) 105.