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Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **67 (1994)**

Heft 7

PDF erstellt am: **12.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-116670

Helv Phys Acta
Vol. 67 (1994)

0018-0238/94/070745-08$1.50+0.20/0

Nonmetricity Driven Inflation

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(12.XI.1994)

Abstract. A metric-affine gravity is examined to generate inflation in cosmology. Using explicit examples it is shown that the dynamical Weyl-type nonmetricity can play the role of a vector inflaton field governing a chaotic-type inflation. Within this approach it seems possible to understand why, at the present epoch, the nonmetricity must be so small, as implied by observations.

1 Introduction

Modern theoretical understanding of evolution of the universe is based on cosmological applications of Einstein's general relativity (GR). In this way, in 1922, A.A.Friedmann derived equations describing an expanding universe [1]. More recently A.Guth [2] suggested that, before the Friedmann stage, the universe had passed through an inflationary stage during which its size was growing very rapidly. The inflationary scenario of evolution of the universe is accepted now by many cosmologists, mainly because it solves many cosmological problems in an elegant way. On the other hand, while solving cosmological problems, inflation produces new, genuine, problems, the main of which is the nature of the field managing the process of inflation (the inflaton field). The original model [2] (called "old inflation") uses the Higgs field of $SU(5)$ theory as the inflaton field. This theory is known to fail because there is no end for the process of inflation [3]. This difficulty was avoided in "new" inflation models [4, 5] by selecting an appropriate shape of the effective potential for the hypothetical inflaton field. Soon after, it was understood that inflation does not need to occur only in very special field theories. A so-called chaotic inflationary scenario [6] shows that, if the field evolves slowly compared to the Hubble parameter, then inflation inevitably arises. This scenario utilizes some minimally coupled scalar field to produce inflation. However, as was shown by Chernikov and Tagirov [7] and recently by Sonego and Faraoni [8], such a field has no physical sense in curved space-time. Moreover, they have shown that, in curved space-time, the physical field is the conformally coupled scalar field. Just this field has valid quasiclassical limit and other good properties. Therefore with a scalar inflaton we find ourselves in a paradoxical situation: the physical conformally coupled scalar field could not generate inflation, and if inflation takes place it inevitably is caused by the unphysical minimally coupled scalar field. Therefore a scalar inflaton is not an appropriate cause of inflation.

Investigating the sources of inflation Ford [9] have shown that inflation could be generated by a vector material field as well. However the nature of this field remains unclear.

In search of an appropriate inflaton field one may go beyond the framework of GR. For instance, one can generalize Einstein's gravity adding a term proportional to R^2 to the Hilbert action [10], or using non-minimal coupling terms [11], or exploiting induced gravity models [12]. In order to obtain an inflationary cosmology one can also use the Jordan-Brans-Dicke theory which contains a natural scalar field representing the time varying gravitational constant [13]. Here we shall not discuss advantages and disadvantages of these approaches referring the reader to the article [14] and references therein. Instead we shall go beyond the framework of GR by enlarging its geometrical background.

It is well known that Riemannian space is a very special case of general metric-affine manifold which is characterized, besides the curvature, by torsion and nonmetricity. In order to obtain Riemannian space from general metric-affine space one must impose two essential restrictions: One has to require that the metric is compatible with the connection (this is a constraint imposed on the metric), and that the connection is symmetric (this is a constraint imposed on the connection). These constraints seem to be too artificial. At the microscopical level there are no physical reasons for the existence of Riemannian structure on the space-time continuum. The existence of the locally Lorentz metric is guaranteed by special relativity while the existence of an affinity is guaranteed by the weak equivalence principle. Hence the existence of the above two objects indicates that, in general, the space-time continuum is a metric-affine manifold. It should be noted that the role of the torsion is rather well understood now (see, e.g. [15, 16] and references therein) but the role of nonmetricity remains unclear (see, however, [17, 18]). There are no sources for nonmetricity for usual matter. Moreover, the presence of nonmetricity at the classical level would contradict the observation data. Hence the role played by nonmetricity, as consider some physicists [18, 19], may be important at the level of microscopic physics and at very high energies. Thus, if such high energies are achievable at the early stage of evolution of the Universe, then the role of nonmetricity becomes fundamental there.

The aim of this paper is to show that in metric-affine gravity there is a natural dynamical field (namely, the Weyl vector field) which can generate and govern the process of inflation in cosmology. Starting with a Hilbert-type Lagrangian we generalize it to account for the nonmetrical properties of space-time completely. For the special choice of nonmetricity (in the Weyl form) the equations of motion following from the Lagrangian we find resemble Einstein gravity with a vector material field as a source. Using an explicit example we demonstrate that in the metric-affine gravity the Weyl nonmetricity can generate and govern the process of inflation and becomes sufficiently small at the end of it.

It is interesting to note that in our approach the question "why is our space-time Riemannian" may have a clear answer: the nonmetrical properties of space-time could have produced an inflationary stage of evolution of the Universe which, in its turn, could have "eaten" nonmetricity, so that at the present epoch it is equal to zero with a high degree of accuracy.

2 Gravity in a Metric-Affine Space-Time

In this section we introduce basic geometrical quantities and notations needed in further considerations. For mathematical details and more deep physical justification we refer the reader to our recent paper [20].

We shall work with a metric-affine space-time with no constraints imposed on the metric $g_{\mu\nu}$ or on the connection $\Gamma^{\lambda}{}_{\mu\nu}$, so that $g_{\mu\nu}$ and $\Gamma^{\lambda}{}_{\mu\nu}$ are completely independent gravitational variables. The nonmetricity tensor $W_{\lambda\mu\nu}$ is defined by

$$W_{\lambda\mu\nu} = \nabla_{\lambda} g_{\mu\nu}. \tag{2.1}$$

It is convenient to split $W_{\lambda\mu\nu}$ in the following way

$$W_{\lambda\mu\nu} = \bar{W}_{\lambda\mu\nu} + 2W_{\lambda} g_{\mu\nu}, \tag{2.2}$$

where W_{λ} is proportional to the trace of the nonmetricity tensor and $\bar{W}_{\lambda\mu\nu}$ is its traceless part

$$W_{\lambda} = \frac{1}{8} W_{\lambda\rho}{}^{\rho}; \quad \bar{W}_{\lambda\rho}{}^{\rho} = 0. \tag{2.3}$$

Using (2.1) one can represent the full affine connection $\Gamma^{\mu}{}_{\nu\rho}$ in the form

$$\Gamma^{\mu}{}_{\nu\rho} = \left\{ {}^{\mu}{}_{\nu\rho} \right\} + S^{\mu}{}_{\nu\rho} - S_{\nu}{}^{\mu}{}_{\rho} - S_{\rho}{}^{\mu}{}_{\nu} + \frac{1}{2}(W^{\mu}{}_{\nu\rho} - W_{\nu}{}^{\mu}{}_{\rho} - W_{\rho}{}^{\mu}{}_{\nu}), \tag{2.4}$$

where $S^{\mu}{}_{\nu\rho}$ is the torsion tensor

$$S^{\mu}{}_{\nu\rho} \equiv \Gamma^{\mu}{}_{[\nu\rho]} = \frac{1}{2}(\Gamma^{\mu}{}_{\nu\rho} - \Gamma^{\mu}{}_{\rho\nu}). \tag{2.5}$$

The curvature tensor $R^{\alpha}{}_{\beta\mu\nu}$ is defined according to

$$R^{\alpha}{}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}{}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}{}_{\beta\mu} + \Gamma^{\alpha}{}_{\lambda\mu}\Gamma^{\lambda}{}_{\beta\nu} - \Gamma^{\alpha}{}_{\lambda\nu}\Gamma^{\lambda}{}_{\beta\mu}. \tag{2.6}$$

Let us define the Ricci tensor $R_{\mu\nu}$ by the formula

$$R_{\mu\nu} = \frac{1}{2}(R^{\sigma}{}_{\mu\sigma\nu} + R_{\mu\sigma\nu}{}^{\sigma}), \quad (R_{\mu\nu} \neq R_{\nu\mu}), \tag{2.7}$$

and the segmental curvature tensor $\Omega_{\mu\nu}$ by

$$\Omega_{\mu\nu} \equiv \frac{1}{4}R^{\sigma}{}_{\sigma\mu\nu} = \partial_\nu W_\mu - \partial_\mu W_\nu. \tag{2.8}$$

Now, let us turn to the choice of the gravitational Lagrangian. As is well known, [21] the main problem in the metric-affine gravity with the Hilbert type Lagrangian

$$L = \alpha R; \quad \alpha = M_p^2/16\pi \tag{2.9}$$

(M_p is the Planck mass) is its projective invariance, i.e., this Lagrangian is invariant under the transformations

$$\Gamma^{\lambda}{}_{\mu\nu} \longrightarrow \Gamma^{\lambda}{}_{\mu\nu} + \delta^{\lambda}_{\mu}\lambda_\nu; \quad g_{\mu\nu} \longrightarrow g_{\mu\nu}. \tag{2.10}$$

As a consequence, four degrees of freedom associated with the Weyl vector W_μ remain completely undetermined by the field equations obtained from this Lagrangian. It is well known [21] that for a viable gravitational theory the projective invariance must be broken. There are several ways to do this. Remember that in GR this problem is solved by imposing the metric condition $\nabla_\lambda g_{\mu\nu} = 0$ that implies $W_\mu = 0$. Here we break the projective invariance by including terms proportional to the square of the Weyl vector $W_\mu W^\mu$ into the gravitational Lagrangian. The total gravitational Lagrangian has the form

$$L = \sqrt{-g}\left(\alpha R + \frac{k}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} + U(\xi)\right) \tag{2.11}$$

where k is a dimensionless constant, $\xi = W_\mu W^\mu$ and $U(\xi)$ is a function (a "potential") which is not projectively invariant. Such form of the Lagrangian could arise as a result of a spontaneous breakdown of scale invariance in the initially conformally invariant theory in a metric-affine space-time (see [20]). In any way we can regard the function $U(\xi)$ as an effective potential analogous to that arising for a scalar field in modern cosmology. Equations of motion following from this Lagrangian can be represented, after some transformations, in the form

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\widetilde{R} = -\frac{1}{2\alpha}T_{\mu\nu}; \tag{2.12}$$

$$\widetilde{\nabla}_\sigma \Omega^{\sigma}{}_{\nu} + 2U'(\xi)W_\nu = 0; \tag{2.13}$$

$$\Gamma^{\mu}{}_{\nu\lambda} = \{^{\mu}{}_{\nu\lambda}\} - \delta^{\mu}_{\nu}W_\lambda; \tag{2.14}$$

$$S_{\mu\nu\lambda} = g_{\mu[\nu}W_{\lambda]}; \tag{2.15}$$

$$\bar{W}_{\lambda\mu\nu} = 0. \tag{2.16}$$

where the tilde above a letter denotes the ordinary Riemannian part of the corresponding geometrical object, and the prime denotes the derivative with respect to ξ. The stress-energy tensor $T_{\mu\nu}$ is equal to

$$T_{\mu\nu} = k\left[\Omega_{\mu}{}^{\lambda}\Omega_{\nu\lambda} - \frac{1}{4}g_{\mu\nu}\Omega_{\alpha\beta}\Omega^{\alpha\beta}\right] - g_{\mu\nu}U + 2U'W_\mu W_\nu. \tag{2.17}$$

As is seen from the above equations a remarkable feature of the theory with the Lagrangian (2.11) is that the Weyl vector here plays the role of a source of the Riemannian part of the curvature. On the other hand, just the Riemannian part of the connection governs the dynamics of the Weyl vector field W_μ, as it is seen from Eq. (2.13). Due to this equation there is no place for the projective invariance. One may consider this theory to be equivalent to usual GR with some external massive vector field. But an essential difference is that here this field is internal and is a part of the full nonmetrical connection, as it is seen from Eq. (2.14). Moreover, this field also determines the torsion properties of the metric-affine space-time via the algebraic relation (2.15).

3 Nonmetricity as a Cause of Inflation

Let us consider what cosmological consequences follow from the theory described by the Lagrangian (2.11). The nonmetrical properties essential for cosmology are contained in the stress-energy tensor (2.17) of the Weyl vector field. In general the stress tensor of this field need not be isotropic, so we may assume that space-time will not be Friedmann-Robertson-Walker type. Instead, we may take an anisotropic metric. Let us take a Bianchi type-I metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2) + b^2(t)dz^2. \tag{3.1}$$

Let W_z be the only nonzero spatial component of the Weyl vector field. We shall be interested in homogeneous solutions, so that $W_\mu = W_\mu(t)$. This leads to the fact that in the metric (3.1) Eq. (2.13) for W_μ implies that $W_t = 0$ for such solutions, and we have for the only nonzero component W_z the following equation:

$$\ddot{W}_z + \left[2\frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right]\dot{W}_z + \frac{2}{k}U'W_z = 0. \tag{3.2}$$

The gravitational equations (2.12) for this metric are:

$$2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}^2}{a^2} = \frac{1}{2\alpha}\varrho, \tag{3.3}$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\ddot{b}}{b} = -\frac{1}{2\alpha}p_x = -\frac{1}{2\alpha}p_y, \tag{3.4}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{2\alpha}p_z \tag{3.5}$$

with the conservation law

$$\dot{\varrho} + \left[2\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right]\varrho + 2\frac{\dot{a}}{a}p_x + \frac{\dot{b}}{b}p_z = 0, \tag{3.6}$$

where the energy density ϱ and pressures p_x, p_z for the Weyl field are

$$\varrho = \frac{k}{2}\frac{\dot{W}_z^2}{b^2} + U, \tag{3.7}$$

$$p_x = \frac{k}{2}\frac{\dot{W}_z^2}{b^2} - U, \tag{3.8}$$

$$p_z = -\varrho + 2\xi U'. \tag{3.9}$$

Let us note that different inflation scenarios within the framework of GR for a material vector field have been considered by Ford [9]. In our approach only a chaotic-type scenario is acceptable because in this case the nonmetricity field tends to zero at the end of inflation. Thus consider the case when initially W_z was nonzero and slowly varying field (so that one can neglect the time derivative terms in (3.7)-(3.9)) but at late times it evolves toward zero. To obey these requirements the potential $U(\xi)$ must have a minimum at $\xi = 0 (\xi = W_z^2/b^2)$. Moreover it must be sufficiently flat for large ξ in order that the duration of inflation would be sufficiently long. With these assumptions and supposing that

$$U \gg 2\xi U' \tag{3.10}$$

we obtain $p_x = p_y \approx p_z \approx -\varrho$, and the universe rapidly comes to the quasi-de Sitter stage

$$a(t) = b(t) = e^{Ht} \tag{3.11}$$

with the slowly varying Hubble parameter

$$H = (U/6\alpha)^{1/2}. \tag{3.12}$$

Let us consider an explicit form of a suitable potential $U(\xi)$:

$$U(\xi) = U_0 \ln\left(1 + \frac{m^2\xi}{2U_0}\right), \tag{3.13}$$

where one may consider U_0 as a lower boundary of inflation below which the inequality (3.10) is broken and the potential becomes rapidly falling. The potential (3.13) mathematically has no upper bound. Nevertheless one can consider its physical upper bound

$$U_{max} \approx M_p^4, \tag{3.14}$$

so that inflation takes place in the region

$$U_0 \lesssim U \lesssim U_{max}. \tag{3.15}$$

Due to the high degree of flatness of the potential (3.13) the process of inflation may be sufficiently long and the value of expansion during inflation may be extremely large. However, in order to obtain adequate inflation one needs considerable fine-tuning.

The field W_z during inflation evolves according to the following equation

$$\ddot{W}_z + H\dot{W}_z + \frac{2}{k}U'W_z = 0. \tag{3.16}$$

For large ξ ($\xi \gg U_0 m^{-2}$) $U' \approx 0$, so the field is a slowly (at most linearly) falling function. On the other hand, for small ξ ($\xi \ll U_0 m^{-2}$) one has $U \approx 1/2m^2\xi$ and the field W_z obeys the equation

$$\ddot{W}_z + H\dot{W}_z + \frac{m^2}{k}W_z = 0. \tag{3.17}$$

In the case

$$\lambda^2 = 4\frac{m^2}{k} - H^2 > 0 \tag{3.18}$$

this equation has a solution in the form of damped oscillations which can be written as

$$W_z = A\exp\left(-\frac{1}{2}Ht\right)\sin\frac{1}{2}\lambda(t-B), \tag{3.19}$$

where A and B are two arbitrary constants. Thus one may consider the region $0 \lesssim U \lesssim U_0$ as corresponding to the reheating stage. In order to obtain some quantitative information about this stage one needs to take into consideration the interaction of matter with torsion and nonmetricity. However in the presence of material sources of torsion and nonmetricity the resulting equations will be essentially different from (2.12)-(2.16). Let us note that due to the fact that W_z tends to zero, the stress tensor of the Weyl field vanishes, so the problem of anisotropy at late times does not arise in this scenario.

In the example considered above we have assumed that inflation goes with the same rates in all directions ($a(t) = b(t) = \exp Ht$). But there is no need for this assumption in cosmology. All that is needed is that the universe expands by the factor greater than 10^{28} in all directions. However this process may go anisotropically, i.e., with different rates along different axes but under the condition that at later stages the anisotropy disappears. The process of anisotropic inflation for the material vector field was considered in detail by Ford [9]. It may be relevant also in the case of inflation driven by vector nonmetricity.

4 Conclusion

As we have seen from the above consideration, nonmetricity in its Weylian form could have played the role of geometrical inflaton field in metric-affine cosmology. By the end of the process nonmetricity disappears, so that space-time becomes Riemannian (or Riemann-Cartan type in the presence of material torsion sources). As it is easy to see, the Weyl nonmetricity produces the cosmological term responsible for inflation because the conformal invariance of the theory is broken from the very beginning (a more sophisticated approach was suggested in [20] where the conformal invariance is preserved but the de-Sitter stage nevertheless exists). However the Weylian form of nonmetricity is not unique. In the general case the nonmetricity tensor $W_{\lambda\mu\nu}$ can describe spin-2 and spin-3 fields [18]. Thus the question what would happen, if the nonmetricity were of a more general form than the Weylian one, needs further investigation.

The other problem concerns the unknown nature of the inflationary potential $U(\xi)$. This is a common problem in contemporary cosmology, independently of the origin of the inflaton field. The presence of the potential $U(\xi)$ is somewhat an *ad hoc* assumption, and a dependence of inflationary dynamics on the precise shape of $U(\xi)$ seems to be a further weakness of inflationary cosmology in general. One may only hope that in the framework of metric-affine gravity with a more general form of nonmetricity this problem may be resolved.

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