Inflationary expansion driven by particle decay and dissipative stress

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Inflationary Expansion Driven by Particle Decay and Dissipative Stress

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Abstract.   The combined effect of decay of heavy bosons into light particles and the matter-radiation interaction may induce a short period of inflationary expansion along with a temperature increase around the baryogenesis era of the Universe.

1   Basic assumptions and numerical analysis

As it has been recently shown [1] change in the number of particles by either direct production or decay of pre-existing particles bears a non-neglible impact on the dynamics and thermal evolution of the early Universe. This may be brought about by the decay of a nonrelativistic fluid into a relativistic one (as for instance the electron-positron annihilation [2], and the decay of heavy bosons into quark and leptons [3]) or the creation of particles and/or strings out of the quantum vacuum [4]. As we will see the combined action of particle decay and a moderate dissipative bulk stress can drive a period of inflationary expansion with a brief increase in the radiation temperature. Let us consider a FRW universe with flat space sections filled with a nonrelativistic fluid made up of massive particles, $\rho_1 \simeq n_1 M$, $P_1 \simeq 0$; and a radiation-like fluid (photons, neutrinos and light particles in general) with

$\rho_2 = (\pi^2/30)g_*T_2^4$, $P_2 = \rho_2/3$, where g_* stands for the total number of relativistic degrees of freedom, which we will consider constant during the period of interest. The former fluid decays spontaneously into the latter with constant rate $\Gamma_1 = \dot{N}_1/N_1$, thereby $\Gamma_2 = \dot{N}_2/N_1$. Here $N_i = n_i R^3$ $(i = 1, 2)$, n_i being the particle number density of the corresponding fluid, R the scale factor of the Robertson-Walker metric and Γ_1 and Γ_2 negative and positive constant rates respectively, such that $\Gamma_1 + \Gamma_2 > 0$. From the above equations it follows that the number of radiation-like particles grows up with time according to

$$N_2 = N_{20} + N_{10}\frac{\Gamma_2}{|\Gamma_1|}(1 - e^{\Gamma_1 t}), \tag{1.1}$$

where the "zero" subindex refers to the time at which the nonrelativistic fluid starts to decay. Let us further assume that both fluids interact with one another and that their interaction is phenomenologically described by a nonequilibrium bulk stress

$$\pi = -\zeta\theta \quad (\theta \equiv 3H, \quad H \equiv \dot{R}/R) \tag{1.2}$$

which for expanding universes tends to decrease the overall pressure. Dissipative stresses like (2) are bound to arise whenever matter and radiation interact with each other. The corresponding phenomenological coefficient $\zeta(\geq 0)$ has been studied from different standpoints: macroscopic [5], kinetic [6] as well as from thermodynamic fluctuation theory [7]. This quantity can be approximated by [8]

$$\zeta = \alpha\rho_2 H^{-1}, \tag{1.3}$$

This expression takes into account that the mean interaction time between particles must be shorter than the expansion time of the fluid as a whole H^{-1}, for any hydrodynamic description to be valid.

Due both to the variation in the particle number density and the heat generated by the dissipative pressure, the radiation temperature evolves according to [9]

$$\frac{\dot{T}_2}{T_2} = \left(\frac{\partial P_2}{\partial \rho_2}\right)_{n_2}\frac{\dot{n}_2}{n_2} + \frac{\zeta}{T_2(\partial\rho_2/\partial T_2)_{n_2}}\theta^2. \tag{1.4}$$

In writing this we have implicitly assumed that the light particles can be described as a fluid right after their creation and that they get thermalized very quickly. With the help of the above expressions the Friedmann equation $H^2 = (8\pi G/3)(\rho_1 + \rho_2)$ can be written as [8]

$$\begin{aligned}
H^2 &= \frac{8\pi G}{3}\{n_{10}M\left(\frac{R_0}{R}\right)^3 e^{-|\Gamma_1|t} \\
&\quad + \frac{\pi^2}{30}g_*\left(\frac{R_0}{R}\right)^{4(1-9\alpha)}[1 + \eta_0\Gamma(1 - e^{-|\Gamma_1|t})]^{4/3}\}
\end{aligned} \tag{1.5}$$

where $\eta_0 \equiv n_{10}/n_{20}$ whereas $\Gamma \equiv \Gamma_2 |\Gamma_1|^{-1}$ stands for the number of radiation-like particles produced in a single decay, i.e. the decay multiplicity. In general this equation does not

admit analytical solutions. However, differentiating (6) with respect to t it can be easily shown that one will have $\ddot{R}(t = 0) > 0$ (initial generalized inflation) if $\alpha > \alpha^*$, where

$$\alpha^* = \frac{1}{36H_0}\{(2 + \frac{\rho_{10}}{\rho_{20}})H_0 + \frac{\rho_{10}}{\rho_{20}} \mid \Gamma_1 \mid -\frac{4}{3}\Gamma_2\}, \tag{1.6}$$

and $\ddot{R}(t = 0) < 0$ otherwise.

To find out the general behavior of the scale factor, the comoving Hubble length $1/(HR)$, and the radiation temperature we have solved numerically equation (5) for different choices of the parameters entering it and used the corresponding solutions in the integrated expression of equation (4). Figures 1 through 3 illustrate the evolution of $R(t)$, $(HR)^{-1}$ and $\chi(t) \equiv T_2(t)/T_{20}$, respectively, for $\mid \Gamma_1 \mid = 1$, $\Gamma = 3$, $\eta_0 = 1$, $R_0 = 1$ (in this case $\alpha^* \simeq 0.024$), and three different values of α: $\alpha = 0$ (no viscosity), 0.05 and 1/12, respectively. The pattern depends on whether α is higher or lower than α^*. In the first case, as figure 2 clearly shows, there is a generalized inflationary period $(d(HR)^{-1}/dt < 0$, i.e. $\ddot{R}(t) > 0)$ along with an initial increase of the temperature. Notice that the expansion remains inflationary independent of the sign of $\dot{\chi}(t)$. The larger the α, the higher the maximum of $\chi(t)$ and the wider the time interval Δt^* for which $T_2(t) > T_{20}$. For high enough α -but still moderate, see curve for $\alpha = 1/12$ in figure 3- $\Delta t^* > \mid \Gamma_1 \mid$. In the second case $(\alpha < \alpha^*)$ -curves with $\alpha = 0$- the expansion results non-inflationary from the start and the radiation temperature decreases monotonously.
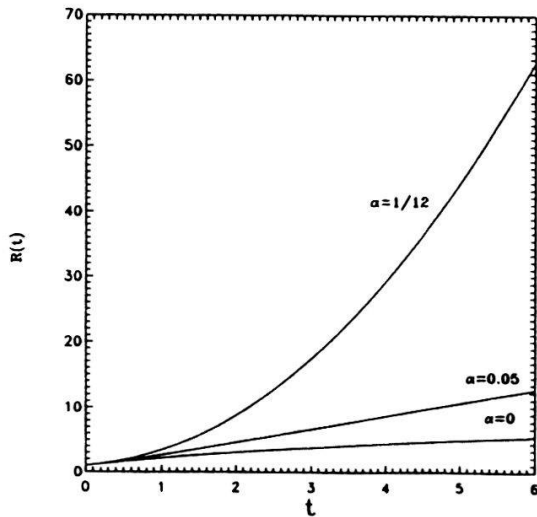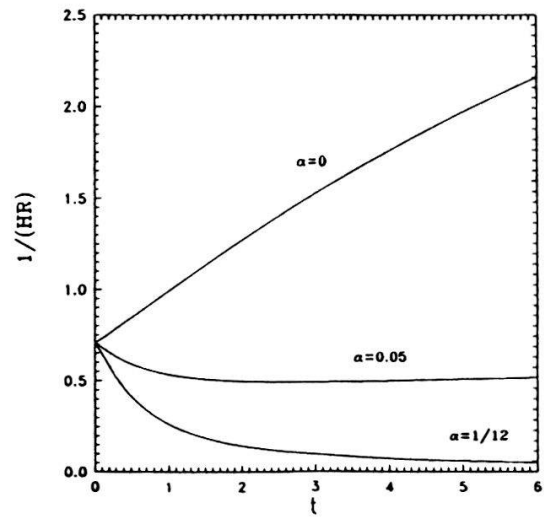

Figure 1

Figure 2

From the acceleration equation

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + 3P_{eff}), \tag{1.7}$$

one may realize that the dissipative bulk pressure alone -equations (2) and (3)- cannot account for the inflationary period as $\ddot{R}/R$ results negative if one takes the effective pressure exclusively as $P_{eff} = P_1 + P_2 + \pi$. This period must be explained as the effect of the combined

action of the bulk pressure and the pressure associated with particle production [1]; i.e. the latter -which has to be included in the *effective* pressure- is so high at the beginning of the decay, that added to π, it overpowers the energy density ρ and the hydrostatic pressures taken together. Alternatively this can be viewed in the following way: at the beginning of the decay $\dot{n}_2$ results very high, and so the radiation temperature increases very quickly and the Hubble factor increases so much that for some time $H^2 > | \dot{H} |$. Later on, because of the sharp decrease in the number of massive particles, $\dot{n}_2$ gets much lower and the expansion ceases to be inflationary. The bulk stress π adds positively, of course, to the rate of expansion.
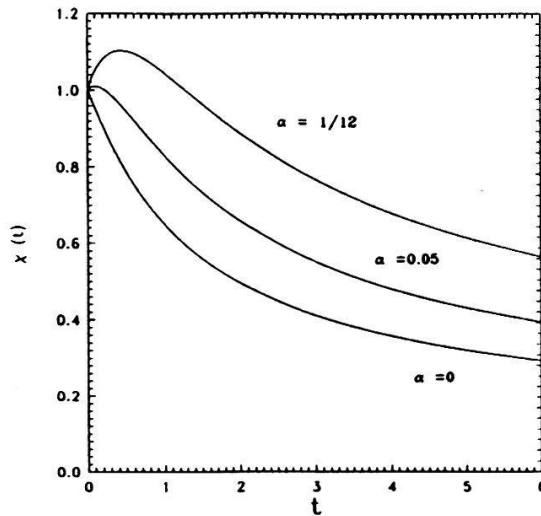

Figure 3

2   Concluding remarks

Due to the limitations of this two-fluid model, mainly because of the assumption $\alpha = constant$, its validity range is restricted to a time interval no much greater than, say, five times $| \Gamma_1 |$. Once this time has elapsed and the massive fluid essentially decayed into the light one, the bulk stress should accordingly be set to zero, and so $R \propto t^{1/2}$ and $T \propto R^{-1}$. However, in this "canonical" evolution the temperature is bound to drop below the rest mass of one or more of the light particles - which initially behaved like radiation because of the high ambient temperature - and a fresh dissipative bulk stress will set in as long as these particles interact with the massless ones, e.g. photons and perhaps neutrinos. It is generally believed that particle decay is unable by itself to drive inflation, and that the bulk pressure can neither, for it should take unrealistic high values. However, as it has turned out, the combination of particle decay and a moderate dissipative bulk stress may render the initial phase of the decay inflationary with a brief increase in the radiation temperature. Obviously for higher multiplicities the inflation will be stronger. Since the quantity $ln[(HR)_f/(HR)_i]$ is lower than 70 in all the cases considered here, this inflation cannot solve the horizon, flatness and smoothness problems of the standard big-bang scenario [10]. These are probably solved

by some version of the ordinary scalar driven inflation theory at the GUTs era [3]. The former, however, must have occurred some time after the latter and prior to the nucleosythesis age -perhaps at the baryogenesis period- and may have helped to dilute undesired relics left over by the phase transitions at GUTs scales [3], [11]. We are indebted to Winfried Zimdahl for conversations and to Tahar Melliti and Vicenç Méndez for their kind computational assistance. One of us (DP) is deeply grateful to the members of the *Laboratoire de Gravitation et Cosmologie Relativistes* for warm hospitality and the *CNRS* for financial support.

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