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Objekttyp: **Article**

Zeitschrift: **Mitteilungen / Vereinigung Schweizerischer Versicherungsmathematiker = Bulletin / Association des Actuaires Suisses = Bulletin / Association of Swiss Actuaries**

Band (Jahr): **70 (1970)**

PDF erstellt am: **30.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-967023

A Note on Profit Margin and Insurance Market Capacity

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1. The rating or pricing of large risks offers very interesting problems. The purpose of this note is neither to discuss all the aspects connected with the measuring of the risk, that is to try to estimate the future pure loss cost from too little or insufficient statistical information, nor to consider the impact of loss prevention measures on this pure loss cost or the net risk rate. In particular the note will not deal with the size factor as a tariff argument. This factor has up to now been rather overlooked in practice though it seems to be an established fact, particularly in Fire, that the loss frequency strongly increases with the size, and it is questionable whether the average degree of damage decreases quickly enough to counterbalance this increase.

2. We will not try to discuss the reasons why it has up to now been possible for the policy holders of large risks to ask for a ,,quantity discount'', such a discount being in most cases unjustified. In this connection we would like to point out that reinsurers, in spite of often having to carry the major part of the risk, have up to now only had a marginal influence on the fixing of the rates.

3. In the following we shall thus leave all the problems of risk measurement aside and tacitly assume that they have been or can be solved.

We shall concentrate on finding some simple and generally applicable rules to fix the additional price which should be generated by size.

4. Our starting point will be an assumed rational insurance and reinsurance market which in its functioning differs from the present one. We hope, however, that the future actual market will develop towards our concepts.

5. In our market, the parties involved are assumed to act in a rational and objective way. It is further assumed that the division of a risk is made with the purpose of producing a minimum price for the insured

within the framework of the conditions of the rational market. This minimum price will for large risks lie substantially above the rates of the present market.

6. The uncertainty in fixing the expected pure loss cost (E) of a treaty or a risk leads us to add a factor proportional to the dispersion (σ) of the claims amount.

If an insurer looks upon a treaty as marginal in relation to the already existing portfolio and wants to be correct in the pricing, taking into consideration his capital and inner reserves, then under simplifying assumptions an additional loading proportional to the variance (σ^2) would come out. Details on this point are to be found in my paper „Some Aspects on Reinsurance Profits and Loadings" which was delivered to the ASTIN-Colloqium in Berlin in 1968.

7. In practice it might be advisable to use a combination of a σ and a σ^2-loading, in other words the price Π net of commissions and possible premium-proportional loadings is

$$\Pi = E + b\sigma + c\sigma^2 \tag{1}$$

where b has the dimension 0 and c the dimension -1.

8. In accordance with para. 3, we shall assume that E is known, and abstain from a σ-loading, i.e. $b = 0$.

9. For each risk there exists an M equal to the insurance amount. Alternatively M could be defined as the PML, in which case this PML would have to be fixed in a very conservative way.

10. We can always introduce a q defined by the relation

$$E = qM .$$

Under simplifying assumptions – the number of claims distributed according to the Poisson Law – we have

$$\sigma \leqq \sigma \max = \sqrt{q}\, M$$

and

$$\sigma^2 \leqq \sigma^2 \max = q M^2 .$$

11. Let us in the following use the approximation

$$\sigma^2 = q M^2 .$$

We thus have

$$\Pi(M) = E + c q M^2 = \tag{2}$$

$$= E(1 + cM) =$$

$$= qM(1 + cM) .$$

12. For a big reinsurer c might be of the order of

$$c = 0.05/\text{mio Sw.frs.}$$

The relation between allocated capacity and desired profit margin (related to the expected pure loss cost E) then becomes:

Capacity M in millions of Sw. frs.	Desired profit margin in per cent
2	10
5	25
10	50
20	100
40	200

In other words the reinsurer is prepared to cover 20 million Sw. frs. if the profit expectation is equal to the expected pure loss cost.

13. Let us accept the pricing formula (2)

$$\Pi(M) = E(1 + cM)$$

and study its consequences. Let us first look into the case where the ceding company is only working with *one* reinsurer.

If the ceding company should decide not to reinsure at all, its minimum price for the risk should be

$$\Pi_c(M) = E(1 + c_c M) .$$

If the reinsurer covers the entire risk, his minimum price should be

$$\Pi_r(M) = E(1 + c_r M)\,.$$

Here c_c and c_r can be understood as price factors for capacity which presumably have been decided by the respective boards. These factors reflect the risk aversion. In general it is to be assumed that c_c is bigger than c_r. No assumption on this point is however necessary for the following.

14. Let us assume that the ceding company cedes and the reinsurer accepts an amount equal to x. The price to cover the risk should then be

$$\begin{aligned} \Pi &= \Pi_c(M-x) + \Pi_r(x) = \\ &= q(M-x)\,[1 + c_c(M-x)] + qx[1 + c_r x] = \\ &= q[M + c_c(M-x)^2 + c_r x^2]\,. \end{aligned} \tag{3}$$

When $x = 0$, in which case the ceding company carries the whole risk, (3) reduces to

$$qM(1 + c_c M) = \Pi_c(M)\,.$$

This will in general represent a fairly high price to the insured.

15. Let us seek the value of x which makes the price for the insured a minimum.

Differentiating (3) and putting the derivative equal to 0 we get

$$q[-2c_c(M-x) + 2c_r x] = 0$$

which gives

$$x = \frac{c_c M}{c_c + c_r} = \frac{\dfrac{M}{c_r}}{\dfrac{1}{c_c} + \dfrac{1}{c_r}}$$

and

$$M - x = \frac{c_r M}{c_c + c_r} = \frac{\dfrac{M}{c_c}}{\dfrac{1}{c_c} + \dfrac{1}{c_r}}\,.$$

The corresponding price is

$$\Pi \min = E\left(1 + M \frac{1}{\frac{1}{c_c} + \frac{1}{c_r}}\right).$$

16. It is tempting to put $\frac{1}{c} = w$ and define w as ,,risk willingness''. If $c = 0.05$/mio Sw. frs. then $w = 20$ mio Sw. frs.

The practical interpretation of this is that an insurer or a reinsurer in this case would be prepared to take over an amount up to 20 million Sw. frs. if the expected profit is equal to the pure loss cost. If the expected profit is higher (lower) the capacity is increased (reduced) accordingly.

Our formulas will then become

$$x = \frac{w_r}{w_c + w_r} M,$$

$$M - x = \frac{w_c}{w_c + w_r} M$$

and

$$\Pi \min = E\left(1 + \frac{M}{w_c + w_r}\right).$$

17. We will illustrate the above results by the following example:

$$c_c = 0.10/\text{mio} \qquad \text{i.e.} \qquad w_c = 10 \text{ mio Sw. frs.}$$

$$c_r = 0.05/\text{mio} \qquad \text{i.e.} \qquad w_r = 20 \text{ mio Sw. frs.}$$

and $M = 12$ mio Sw. frs.

We easily see that where there is no reinsurance we have

$$\frac{\Pi_c(M)}{E} = 1 + \frac{12}{10} = 2.2$$

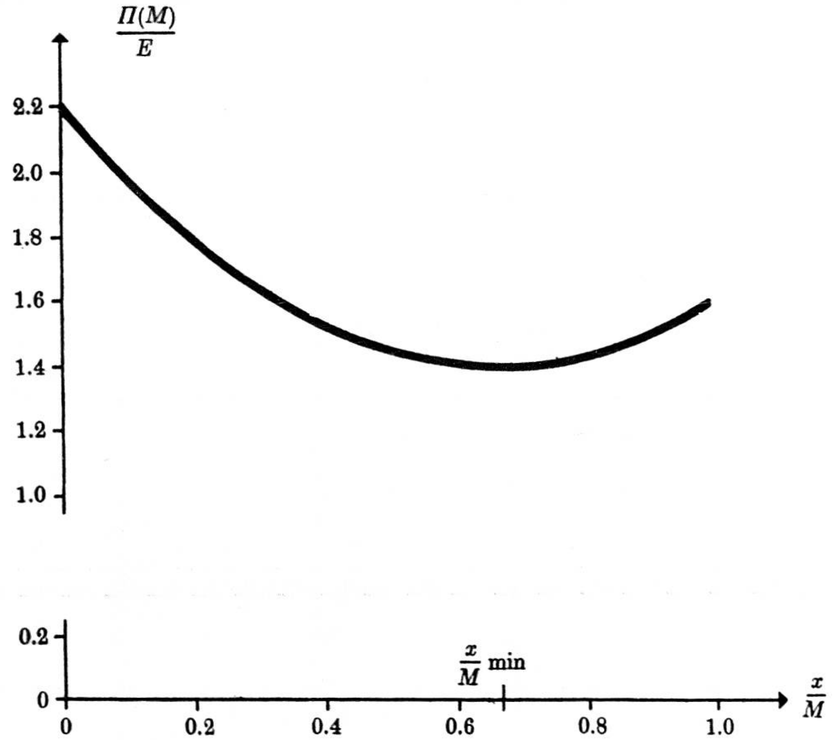
and correspondingly when the reinsurer carries the whole risk we arrive at

$$\frac{\Pi_r(M)}{E} = 1 + \frac{12}{20} = 1.6\,.$$

When the risk is divided in an optimal way between the parties the minimum price is

$$\frac{\Pi \min}{E} = 1 + \frac{12}{10 + 20} = 1.4\,.$$

The price as a function of x is illustrated in the diagram below:



18. The results of the last section illustrate the important services of the reinsurer. The price which has to be charged to the insured if no reinsurance (or co-insurance) facilities exist, is considerably higher than the price which can be offered if reinsurance is available. It is further clear that the price will to a large extent depend on the risk-willingness of the reinsurer. This further underlines the absurdity of the present situation where the ceding company fixes the rate for the insured without consulting the reinsurer who is in most cases going to carry the major burden.

19. Above we have studied the case of only *one* reinsurer. It is obvious that the price to the insured can be further reduced by a wider spread of the risk. This can be done through bringing in further reinsurers or by using retrocession or co-insurance. We will concentrate on the first case. The mathematical treatment will cover the other cases and any combination of them as well.

20. Let us thus assume that n reinsurers are brought into the picture. Each of them is characterised by a risk-willingness

$$w_{r_i} \qquad i = 1 \ldots n .$$

We have to determine the shares x_{r_i},

so that

$$\Pi_c\left(M - \sum_{i=1}^{n} x_{r_i}\right) + \sum_{i=1} \Pi_{r_i}(x_{r_i})$$

becomes a minimum.

When differentiating with regard to each x_{r_i}, we obtain a set of n equations which will give the solution.

$$x_{r_i} = \frac{M\,w_{r_i}}{w_c + \sum_{i=1}^{n} w_{r_i}} \qquad i = 1 \ldots n$$

and
$$\Pi \min = E\left(1 + \frac{M}{w_c + \sum_{i=1}^{n} w_{r_i}}\right)$$

21. Suppose that the entire available market consists of k reinsurers (or co-insurers) which are characterised by $w_1, w_2 \ldots w_k$

The total capacity M_r of the market will depend upon the profit margin.

Using the above results and denoting the relative profit margin by ε it is easily seen that

$$M_r = \varepsilon \sum_{i=1}^{k} w_{r_i}$$

We shall go further into this problem in section 24. If the available market is given and cannot be extended the capacity can still be strongly increased if a higher profit margin is allowed for. This coincides with practical observations. In the above simplified model the capacity increases in direct proportion to the profit margin.

22. The economic and technical development will lead to a considerable increase in the *demand* for capacity. From the above it appears that reinsurers (and insurers) should try to define a sensible price for capacity in order to be prepared to meet the increasing demand for capacity. This will often lead to considerably higher rates than those applied at present. I could imagine that for big industries fire rates will come out which are 2 to 5 times the present ones.

23. The industries which will have to pay this increased insurance cost should in principle be able to raise the additional money out of the rationalisation profits which result from the concentration of the manufacturing process. If the factories were divided into minor units, this would result in higher production costs, being only partly counterbalanced by the reduction in exposure and premium.

24. In section 21 we have deduced the total reinsurance market capacity as being

$$M_r = \varepsilon \sum_{i=1}^{k} w_{r_i}$$

where ε stands for the relative profit margin.

If $M > M_r$ and $M - M_r > \varepsilon w_c$ this means that the ceding company will either have to carry more for own account than it really wants or leave the insured partly uncovered or both.

The natural way out of this dilemma is to allow for a higher ε i.e. increase ε to ε_k such that

$$\varepsilon_k \geqq \frac{M}{w_c + \sum_{i=1}^{k} w_{r_i}} . \tag{4}$$

25. We would like to illustrate (4) with an example. Suppose that a big reinsurer has

$$c_1 = 0.05/\text{mio} \qquad \text{or} \qquad w_1 = 20\,\text{mio}.$$

w_1 is thus equal to the capacity available from the reinsurer r_1 if the rate contains a profit margin equal to the expected pure loss cost ($\varepsilon = 1$). The same situation shall hold for all reinsurers r_i, who together may be able to cover under the above-mentioned profit margin 500 million Sw. frs.

	r_i	w_{r_i} in mio Sw. frs.
	r_1	20
	r_2	20
	r_3	15
	⋮	⋮
Total		500

The market can thus absorb a risk of 500 million Sw. frs. when the profit margin is 100% of the expected pure loss cost and 50 million Sw. frs. when the profit margin is only 10%.

Expected profit margin in relation to pure loss cost in per cent (100 ε)	Available capacity in millions of Sw. frs.
5	25
10	50
20	100
40	200
100	500
200	1000
400	2000

Remark

The results obtained are based on a σ^2-loading. With a pure σ-loading ($c = 0$ in formula (1)) it is not possible to deduct a rational market behaviour. If there are only two parties – C and R – a σ-loading would lead to the result that either C or R accepts the whole risk – and this irrespective of the size of the risk.

Conclusions

It is possible to enlarge the above model in various directions in order to increase its realism. Already in this simplified form, however, it leads to conclusions which are in accordance with practical observations. The mathematics are simple and the results are easily understood. It is to be observed that the only parameter to be fixed for a company is $w = \dfrac{1}{c}$.

This is to my mind an advantage.

An alternative starting point is to base the considerations on the more vague theory of utility of money, where we will have considerable difficulties in fixing the parameters in the utility functions of the various companies.

Final comment

The previous considerations about covering large risks appear to be neutral with regard to the type of market organisation.

Though perhaps naturally I have primarily had the conditions in the Western countries in mind, I would guess that the same basic reasoning would be applicable elsewhere.

Summary

Based on some simplifying considerations and the assumption of a rational insurance market, the author deduces a price for capacity, i.e. the addition which should be made to the expected pure loss cost. He also calculates the market capacity for a certain risk as a function of the profit margins in the rates and estimates the "risk-willingness" of the market.

Résumé

Partant de quelques indications simplificatrices et de l'hypothèse d'un marché rationnel d'assurance, l'auteur parvient à un prix pour la capacité, c'est-à-dire au chargement qui doit être ajouté à la charge de sinistre pure attendue. Il calcule aussi la capacité du marché pour un certain risque en fonction de la marge bénéficiaire contenue dans les taux et détermine la «kindunophilie» du marché.

Zusammenfassung

Auf Grund vereinfachender Annahmen entwickelt der Verfasser für einen «vernünftigen» Versicherungsmarkt einen Preis für Kapazität, das heisst den Zuschlag auf der erwarteten, reinen Schadenlast. Er berechnet die verfügbare Marktkapazität für ein bestimmtes Risiko als Funktion der Profitmarge und bestimmt die «Risikobereitschaft» des Marktes.

Riassunto

Basandosi su alcune considerazioni semplificatrici, e sull'ipotesi di un mercato ideale delle assicurazioni, l'autore desume un prezzo per la capacità, ossia il caricamento da aggiungere al costo netto sperato dei sinistri. Egli calcola pure la capacità del mercato per un rischio determinato, quale funzione dei margini di profitto inclusi nei tassi, e la disponibilità del mercato per il rischio.