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Autor(en): **Halmstad, David G.**

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Illustration of the nonlinearity of risk theory under uncertainty in the claim rate parameter

By David G. Halmstad, Ridgefield, Conn.

Let an insurer specify that his unwillingness to lose $\underline{u}$ of surplus be limited to a probability $\underline{\psi}$ on a coverage for which the claim amount distribution is known. Ruin theory methods allow one to solve for the loading required on the coverage to meet this limitation, and such calculations may be carried out on an infinite or a finite time base. In the latter case, however, the number of expected claims must be known.

One form of the dependency of the loading on the number of expected claims in the finite period case is shown in Figure 1. For this illustration, the claim amount distribution is degenerate (a single amount, which may be taken as unity) and the exact amount of loading can be obtained from Seal's equation 4.20[1]. The numerical results corresponding to Figure 1, at $\underline{\psi} = .005$, are given in Table 1 for $\underline{u} = 2, 3, 4$ and 5.

While strict use of risk theory in this "unit claim" case allows an insurer to enjoy his non-ruin guarantee without collecting any premiums at all when

$$\sum_{\varkappa=\lceil \underline{u} \rceil}^{\infty} e^{-\xi} \frac{\xi^{\varkappa}}{\varkappa!} \leq \underline{\psi}, \qquad \text{where } \underline{\xi} \text{ is the net expected}$$

claims and $\lceil \cdot \rceil$ denotes the next integer operator (that is, $\lceil \underline{u} \rceil$ is the least integer greather than or equal to $\underline{u}$), and with charging at most the net expected claims $\underline{\xi}$ as premiums whenever

$$\sum_{\varkappa=\lceil \underline{u}+\underline{\xi} \rceil}^{\infty} e^{-\xi} \frac{\xi^{\varkappa}}{\varkappa!} \leq \underline{\psi},$$

we will assume that an insurer insists on receiving at least the net premiums $\underline{\xi}$. Thus the left portion of each of the four cases illustrated in Figure 1 will be interpreted as if the level of loading is zero.

It is apparent from Figure 1 that the functional dependency of the loading on the expected claims is complicated. To continue our illustration of the effect of an insurer's uncertainty about his claim rate parameter (ξ) on the loading (λ) he needs, we shall assume that

$\lambda_{\underline{u}}(\xi) = \lambda_{\underline{u}} \times \max(0, 1-\beta^{-1} e^{-\beta\xi})$, where $\lambda_{\underline{u}}$ is the infinite period loading corresponding to an initial surplus $\underline{u}$ for a given ruin level $\underline{\psi}$. The parameter β,

[1] H. L. Seal, *Stochastic Theory of a Risk Business*, John Wiley & Sons, New York, 1969.

which determines the point at which the loading λ emerges from the zero line, also depends on the joint values of $\underline{u}$ and $\underline{\psi}$.

Suppose now that the insurer can specify his uncertainty about ξ by a gamma distribution $f(\xi) = \dfrac{\xi^{\alpha-1} e^{-\xi}}{\Gamma(\alpha)}$, the natural conjugate distribution for the parameter ξ of a Poisson distribution. In this case, the expected value of the loading needed by the insurer is

$$E[\lambda(\xi)] = \int_0^\infty \lambda(\xi) f(\xi)\, d\xi = \lambda_{\underline{u}} \int_{(-\ln\beta)\div\beta}^{\infty} \left(1-\beta^{-1} e^{-\beta\xi}\right) \frac{\xi^{\alpha-1} e^{-\xi}}{\Gamma(\alpha)}\, d\xi$$

$$= \lambda_{\underline{u}} \times \left[\frac{\Gamma(\alpha, \{-\ln\beta\}\div\beta)}{\Gamma(\alpha)} - \frac{\Gamma(\alpha, \{-\ln\beta\}\times\{\beta+1\}\div\beta)}{\beta(\beta+1)^{\alpha}\Gamma(\alpha)}\right]$$

where $\dfrac{\Gamma(\alpha, x)}{\Gamma(\alpha)} = \displaystyle\int_x^\infty \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)}\, dy$ is the incomplete gamma function ratio.

Selected values of this expected loading, $E[\lambda(\xi)]$, for various combinations of α and β, are shown in Table 2, and compared with the loading applicable at the expected claim rate, $\lambda(E[\xi])$, all values determined as ratios of the infinite period loading, $\lambda_{\underline{u}}$.

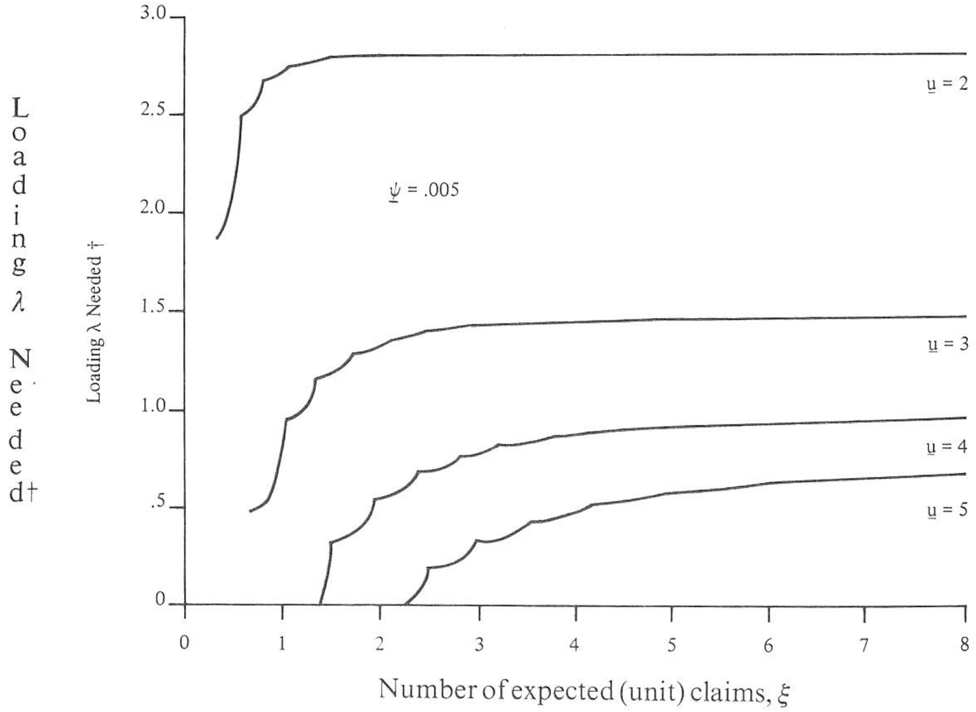

Figure 1: Dependency of loading on number of expected claims

† Loading given as a multiple of expected claims

Table 1

Loading needed with initial capital U for expected [unit] claims ξ

Time ξ	U = 2	U = 3	U = 4	U = 5
0.10				
0.20				
0.30				
0.31				
0.32				
0.33				
0.34	1.95980			
0.35	1.96044			
0.36	1.96204			
0.37	1.96466			
0.38	1.96839			
0.39	1.97330			
0.40	1.97951			
0.41	1.98710			
0.42	1.99622			
0.43	2.00698			
0.44	2.01956			
0.45	2.03413			
0.46	2.05089			
0.47	2.07010			
0.48	2.09202			
0.49	2.11699			
0.50	2.14542			
0.51	2.17777			
0.52	2.21463			
0.53	2.25670			
0.54	2.30489			
0.55	2.36033			
0.56	2.42447			
0.57	2.49924			
0.58	2.50483			
0.59	2.50535			
0.60	2.50627			
0.61	2.50760			
0.62	2.50937			
0.63	2.51161			
0.64	2.51434			
0.65	2.51759			
0.66	2.52139			
0.67	2.52579			

Time ξ	$\underline{U} = 2$	$\underline{U} = 3$	$\underline{U} = 4$	$\underline{U} = 5$
0.68	2.53081	0.48771		
0.69	2.53650	0.48800		
0.70	2.54289	0.48854		
0.71	2.55004	0.48934		
0.72	2.55800	0.49041		
0.73	2.56681	0.49177		
0.74	2.57655	0.49345		
0.75	2.58728	0.49545		
0.76	2.59906	0.49781		
0.77	2.61199	0.50054		
0.78	2.62614	0.50369		
0.79	2.64163	0.50726		
0.80	2.65855	0.51131		
0.90	2.69105	0.58920		
1.00	2.71861	0.83901		
1.10	2.75500	0.96549		
1.20	2.76120	0.99380		
1.30	2.77824	1.06459		
1.37	2.78412	1.16586		
1.38	2.78428	1.17929	0.00803	
1.39	2.78447	1.17936	0.01699	
1.40	2.78469	1.17953	0.02698	
1.41	2.78495	1.17981	0.03818	
1.42	2.78524	1.18018	0.05082	
1.43	2.78558	1.18066	0.06518	
1.44	2.78596	1.18126	0.08164	
1.45	2.78638	1.18197	0.10072	
1.46	2.78684	1.18280	0.12312	
1.47	2.78736	1.18376	0.14990	
1.48	2.78792	1.18485	0.18265	
1.49	2.78853	1.18608	0.22401	
1.50	2.78919	1.18745	0.27864	
1.60	2.79671	1.21077	0.33135	
1.70	2.79768	1.25928	0.34782	
1.80	2.80062	1.29430	0.38676	
1.90	2.80269	1.30126	0.47714	
2.00	2.80345	1.31769	0.54827	
2.10	2.80529	1.34829	0.55659	
2.20	2.80549	1.35833	0.57663	
2.21	2.80552	1.35864	0.57954	0.00017
2.22	2.80556	1.35899	0.58265	0.00160
2.23	2.80560	1.35938	0.58599	0.00316
2.24	2.80564	1.35982	0.58955	0.00484
2.25	2.80569	1.36031	0.59336	0.00666
2.26	2.80575	1.36084	0.59744	0.00863

Time ξ	$\underline{U} = 2$	$\underline{U} = 3$	$\underline{U} = 4$	$\underline{U} = 5$
2.27	2.80581	1.36143	0.60181	0.01076
2.28	2.80587	1.36207	0.60647	0.01305
2.29	2.80595	1.36276	0.61147	0.01553
2.30	2.80602	1.36350	0.61682	0.01820
2.30	2.80602	1.36350	0.61682	0.01820
2.40	2.80667	1.37443	0.67921	0.06049
2.50	2.80680	1.39343	0.68246	0.17471
2.60	2.80715	1.39573	0.69205	0.19751
2.70	2.80732	1.39932	0.71108	0.20510
2.80	2.80742	1.40643	0.74529	0.22179
2.90	2.80761	1.41762	0.76385	0.25487
3.00	2.80763	1.41837	0.76790	0.32565
3.10	2.80770	1.42835	0.77689	0.33274
3.20	2.80776	1.43120	0.79290	0.33878
3.30	2.80778	1.43377	0.81899	0.35142
3.40	2.80783	1.43607	0.82017	0.37469
3.50	2.80784	1.43813	0.82411	0.41769
3.60	2.80785	1.43998	0.83175	0.42531
3.70	2.80788	1.44164	0.84444	0.42965
3.80	2.80788	1.44313	0.85729	0.43858
3.90	2.80789	1.44448	0.85865	0.45429
4.00	2.80790	1.44442	0.86210	0.48093
4.10	2.80790	1.44687	0.86933	0.49283
4.20	2.80790	1.44793	0.87482	0.50258
4.30	2.80790	1.44890	0.87995	0.51177
4.40	2.80791	1.44980	0.88473	0.52041
4.50	2.80791	1.45063	0.88919	0.52856
4.60	2.80791	1.45138	0.89334	0.53624
4.70	2.80791	1.45207	0.89722	0.54350
4.80	2.80791	1.45270	0.90084	0.55037
4.90	2.80791	1.45327	0.90422	0.55689
5.00	2.80791	1.45360	0.90694	0.56235
5.10	2.80791	1.45427	0.91037	0.56901
5.20	2.80791	1.45469	0.91317	0.57467
5.30	2.80791	1.45508	0.91580	0.58007
5.40	2.80791	1.45542	0.91827	0.58522
5.50	2.80791	1.45574	0.92059	0.59014
5.60	2.80791	1.45602	0.92276	0.59485
5.70	2.80791	1.45627	0.92480	0.59935
5.80	2.80791	1.45650	0.92672	0.60365
5.90	2.80791	1.45671	0.92853	0.60776
6.00	2.80791	1.45682	0.93005	0.61148
6.10	2.80791	1.45708	0.93183	0.61548
6.20	2.80791	1.45724	0.93335	0.61911

Time ξ	$\underline{U} = 2$	$\underline{U} = 3$	$\underline{U} = 4$	$\underline{U} = 5$
6.30	2.80791	1.45740	0.93478	0.62258
6.40	2.80791	1.45753	0.93613	0.62592
6.50	2.80791	1.45766	0.93740	0.62911
6.60	2.80791	1.45778	0.93860	0.63218
6.70	2.80791	1.45789	0.93974	0.63512
6.80	2.80791	1.45798	0.94081	0.63795
6.90	2.80791	1.45807	0.94182	0.64066
7.00	2.80791	1.45814	0.94271	0.64318
7.10	2.80791	1.45822	0.94369	0.64578
7.20	2.80791	1.45829	0.94455	0.64819
7.30	2.80791	1.45835	0.94536	0.65051
7.40	2.80791	1.45840	0.94613	0.65274
7.50	2.80791	1.45845	0.94686	0.65488
7.60	2.80791	1.45849	0.94756	0.65695
7.70	2.80791	1.45853	0.94821	0.65894
7.80	2.80791	1.45856	0.94883	0.66085
7.90	2.80791	1.45860	0.94942	0.66270
8.00	2.80791	1.45861	0.94995	0.66440
8.10	2.80791	1.45865	0.95051	0.66619
8.20	2.80791	1.45868	0.95102	0.66785
8.30	2.80791	1.45870	0.95150	0.66944
8.40	2.80791	1.45873	0.95195	0.67097
8.50	2.80791	1.45875	0.95238	0.67244
8.60	2.80791	1.45877	0.95279	0.67385
8.70	2.80791	1.45879	0.95318	0.67521
8.80	2.80791	1.45880	0.95355	0.67650
8.90	2.80791	1.45882	0.95390	0.67774
9.00	2.80791	1.45883	0.95423	0.67892
9.10	2.80791	1.45884	0.95454	0.67997
9.20	2.80791	1.45885	0.95483	0.68097
9.30	2.80791	1.45886	0.95511	0.68193
9.40	2.80791	1.45887	0.95537	0.68284
9.50	2.80791	1.45888	0.95563	0.68372
9.60	2.80791	1.45888	0.95588	0.68458
9.70	2.80791	1.45889	0.95612	0.68541
9.80	2.80791	1.45889	0.95636	0.68622
9.90	2.80791	1.45890	0.95659	0.68703
10.00	2.80791	1.45891	0.95682	0.68782
∞	2.80791	1.45897	0.96118	0.71218

Table 2: *Illustrative differences between*

(a) $\lambda(E[\xi])$, loading calculated at the expected claim rate, and
(b) $E[\lambda(\xi)]$, the expected value of loading needed when uncertainty in the claim rate parameter is recognized.

(a) $\lambda(E[\xi]) \div \lambda_{\underline{u}}$

$\alpha =$	1	2	3	4
$\beta = 0.3$	0	0	0	0
0.5	0	0.26424	0.55374	0.72933
0.7	0.29059	0.64772	0.82506	0.91313

(b) $E[\lambda(\xi)] \div \lambda_{\underline{u}}$

$\alpha =$	1	2	3	4
$\beta = 0.3$	0.00417	0.02412	0.07305	0.15563
0.5	0.08333	0.25441	0.44854	0.61496
0.7	0.24738	0.51894	0.71080	0.82911

Zusammenfassung

Die Einwirkung der Unsicherheit über die erwartete Schadenbelastung auf die Berechnung von Schwankungs-Kostenzuschlägen wird anhand eines einfachen Beispiels erläutert, welches eine exponentielle Kurvenform für die Kostenzuschläge für endliche Zeitabschnitte voraussetzt.

Summary

The effect of uncertainty in the claim rate parameter on an insurer's calculation of fluctuation loadings is discussed and illustrated by a simple case which uses an assumed exponential form for loadings in finite periods.

Résumé

On discute l'effet de l'incertitude dans le paramètre représentant le taux des sinistres sur les calculs des marges de l'assureur. Une illustration est donnée dans un cas simple dans lequel on emploie une forme exponentielle hypothétique pour la marge de securité dans une periode finie.

Riassunto

L'effetto dell'incertezza nel parametro della distribuzione dei sinistri sui calcoli del fattore di carico viene discusso ed illustrato in un caso semplice che prescuppone una forma esponenziale per il fattore di carico nei periodi finiti.