Refund formula in group life insurance

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Introduction

In some countries where the insurance companies are free to act, one can find, besides plans based on the past experience of the group, plans where a part of the excess of the paid premium over the incurred claims and expenses is refunded to the policyholder. Dependent upon the number of assured lives the non-profit premium is increased by a certain percentage. The policyholder undertakes to pay such a loading and the insurer grants a refund depending on the number of lives in the group and the requested premium loading.
With-profit quotations are normally only accepted for at least 500 lives (or 250 lives with an accounting period of 2 years = 500 life-years).
Hereinafter "life-years" shall mean the number of lives in the group accumulated during the accounting period.

1 Model and Definition

The refund formula can generally be written as:

$$\begin{aligned} R &= b\,(agP - c) \quad \text{if } (agP - c) > 0 \\ R &= 0 \qquad\qquad\quad \text{if } (agP - c) \leq 0 \end{aligned} \tag{1.1}$$

with $P =$ net annual premium
$1 - a =$ deduction in per cent for Stop-Loss cover, catastrophe risk, etc.
$c =$ claims during the insurance year
$g - 1 =$ profit loading
$b =$ percentage of profit share

Note: C denotes the random variable of the claims, whereas c are the claims actually experienced during the insurance year.

It is known that for fixed parameters a and g, the percentage of profit share b depends on the number of insured lives and on the distribution of the sums at risk. This is measured by the total claim distribution which depends on the scheme's size. Before starting with the analysis of the claim distribution we first build a model which should help us to understand the computations presented in the following chapters.

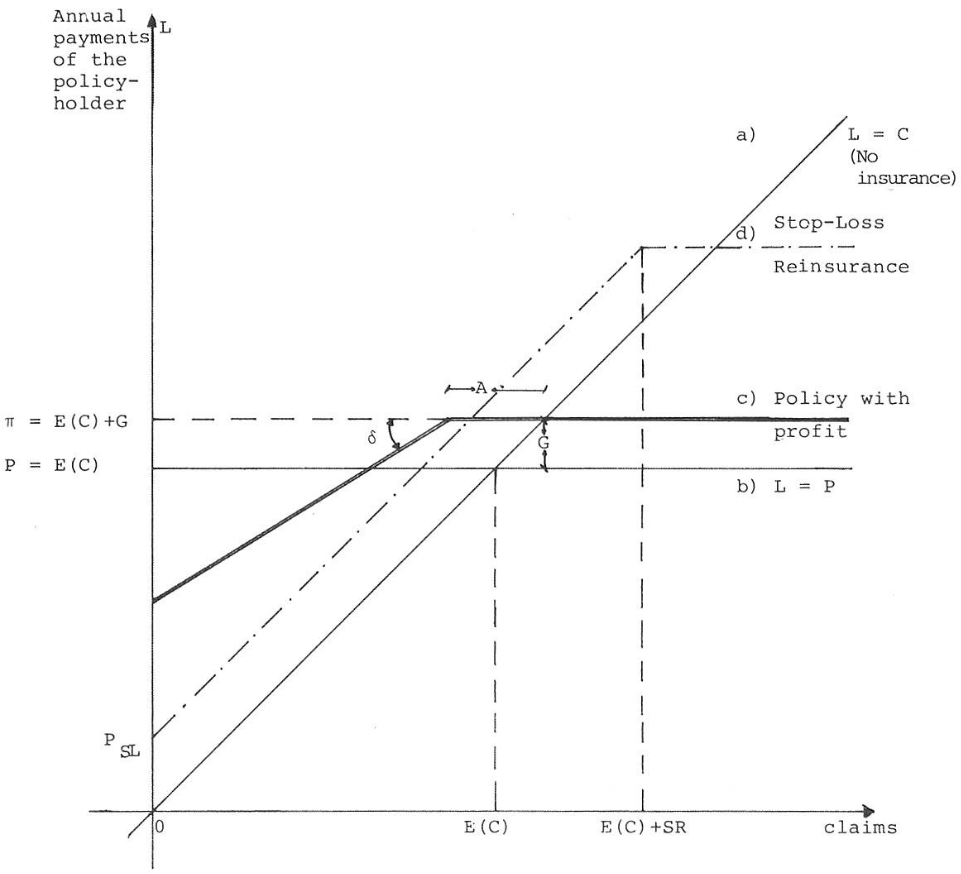
Definition (1.2)

"Principle of equivalence": The amount of premiums required for a given insurance can be determined from the principle that the present value of the net premium must be equal to the present value of the insurance benefits.
Applying the "principle of equivalence" as defined above, the net premium becomes

$P = E(C)$: claims expectation, and the with-profit premium can be written as (1.3)

$\pi = g\ P = g\ E(C).$ (1.4)

Model (1.5)



where: G $= (g-1)\ P$: Loading
A $= (1-a)\ \pi$: Deduction
P_{SL} = Stop-Loss premium
$E(C)+SR$ = Stop-Loss point (retention limit)
L = Annual payments of the policyholder

The model (1.5) shows the annual payments of the policyholder for 4 different kinds of policies.

a) $L=C$ — All the claims that will occur during the year, have to be met by the policyholder.

b) $L=P$ — The policyholder pays a net annual premium P equal to the claims expectation $E(C)$, "principle of equivalence", and he is covered against every claim which will occur during the insurance year.

c) $L=E(C)+G-R$
$\quad =\pi-R$ — In this case the policyholder pays a higher premium, but if the claims do not exceed $\pi-A$, part of the premiums paid will be refunded (R).

d) $L=C+P_{SL}$ — if $c<E(C)+SR$: Stop-Loss.

The policyholder pays a Stop-Loss premium P_{SL} that depends on the retention limit, plus all the claims up to the retention limit $E(C)+SR$.

Following the model above, the refund formula can also be written as:

$$R=\tan\delta\ \{\max[0, (E(C)+G-A)-c]\} \tag{1.6}$$

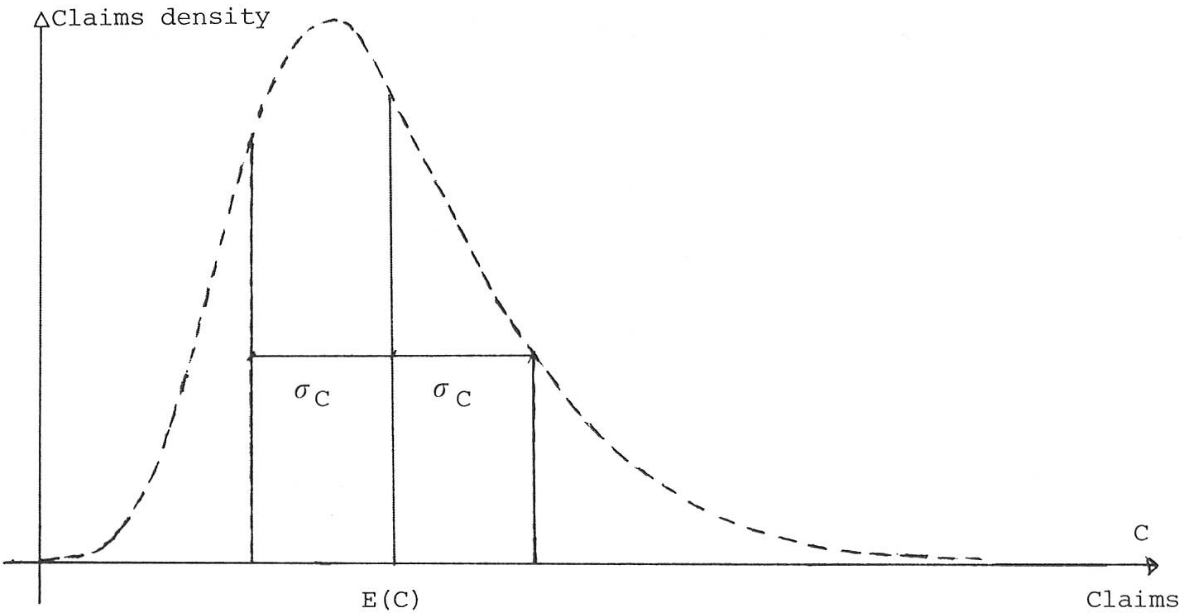
with

$$b=\tan\delta \tag{1.7}$$

Remark: b is normally bounded $0\leq b\leq 1$, but $b>1$ is also acceptable, e.g. if we are asking for an excessively high profit loading or deduction $(1-a)$.

2 The total claim distribution

This chapter deals with the problem of finding the total claim distribution of a basic scheme (10,000 lives). Taking into account a group of 10,000 lives and simulating the claims for a period of one year, we obtain a claims density as follows:



with C = Claims = Risk sum

(For one year risk insurances the risk sum is equal to the sum insured because there are no reserves.)

A good approximation of this empiric density is the density of the log-normal distribution:

$$f(c)=\frac{1}{\sqrt{2\pi}\,\sigma\, c}\, e^{-\frac{1}{2}\left(\frac{\ln c-\mu}{\sigma}\right)^2} \tag{2.1}$$

with
$$E(C)=e^{\mu+\frac{\sigma^2}{2}} =: \mu_C \tag{2.2}$$

$$\operatorname{Var}(C)=(E(C))^2\,(e^{\sigma^2}-1)=:\sigma_C^2 \tag{2.3}$$

(2.2) and (2.3) lead to the following definition of the parameters μ and σ:

$$\sigma_C^2=(\mu_C)^2\,(e^{\sigma^2}-1)\rightarrow\frac{\sigma_C^2}{\mu_C^2}+1=e^{\sigma^2}$$

$$\sigma^2=\ln\left(\frac{\sigma_C^2}{\mu_C^2}+1\right) \tag{2.4}$$

$$\ln \mu_C=\mu+\frac{\sigma^2}{2}$$

$$\mu=\ln \mu_C-\frac{\sigma^2}{2} \tag{2.5}$$

The results of the simulation are*:

$$\mu_C = E(C) = 156.52 \tag{2.6}$$

$$\sigma_C^2 = \mathrm{Var}(C) = 3721 \rightarrow \sigma_C = 61 \tag{2.7}$$

Applying (2.4) and (2.5) we obtain

$$\sigma^2 = \ln\left(\frac{\sigma_C^2}{\mu_C^2} + 1\right) = 0.14161 \tag{2.8}$$

$$\mu = \ln \mu_C - \frac{\sigma^2}{2} = 4.98248 \tag{2.9}$$

3 Correction of the density parameters

The simulation leads us to a claims density function for a group of 10,000 lives (basic group). We are also interested in the claims density of smaller or larger groups (e.g. 1,000; 5,000; 8,000; 15,000; ... lives). If we want to avoid calculations for the simulation of every group, we should be able to change the density parameters in function of the size of the group.

Let Z, H be random variables defined as follows

$$Z = \text{number of claims}$$

$$H = \text{amount of a single claim}$$

with expectations μ_Z, μ_H

and variances σ_Z^2, σ_H^2 respectively.

Assuming that Z follows the Poisson distribution we can write:

$$\sigma_Z^2 = \mu_Z \tag{3.1}$$

(The variance and the expectation of a Poisson variable are identical.)

The claims expectation μ_C can be computed as follows

$$\mu_C = \mu_Z \, \mu_H \tag{3.2}$$

* *Remark*: The results given above are based on a collective of 10,000 lives with a certain distribution of the sums at risk, that could of course be different from that of other schemes of the same size.

and the variance σ_C^2 is

$$\sigma_C^2 = \mu_Z \, \sigma_H^2 + \mu_H^2 \, \sigma_Z^2 \tag{3.3}$$

Putting (3.1) and (3.2) in (3.3) we obtain

$$\sigma_C^2 \underset{\substack{\uparrow \\ (3.1)}}{=} \mu_Z \, (\sigma_H^2 + \mu_H^2) \underset{\substack{\uparrow \\ (3.2)}}{=} \frac{\mu_C}{\mu_H} (\sigma_H^2 + \mu_H^2) \tag{3.4}$$

N = basic number of lives (= 10,000)

N^* = new number of lives

$\bar{S}$ = average sum insured in the basic group

$\bar{S}^*$ = new average sum insured in the group

Hereinafter indicates* the new values.

The expected number of claims and the amount of the single claim are affected by the change of N and $\bar{S}$ in the following way:

$\mu_Z^* = \dfrac{N^*}{N} \mu_Z$ i.e. the expected number of claims is proportional to the increasing (decreasing) number of lives.

$H^* = \dfrac{\bar{S}^*}{\bar{S}} H$ Since in group life insurance the sum insured is in most cases directly related to the annual income, an increase in the salaries all over the group will produce the same increase in the single claims.

The new expectations and variances are

$$\mu_H^* = \frac{\bar{S}^*}{\bar{S}} \mu_H \; ; \; \sigma_H^{2*} = \mathrm{Var}\left[\frac{\bar{S}^*}{\bar{S}} H\right] = \left(\frac{\bar{S}^*}{\bar{S}}\right)^2 \sigma_H^2$$

so that the claims expectation with respect to (3.2) becomes:

$$\mu_C^* = \mu_Z^* \, \mu_H^* = \frac{N^*}{N} \frac{\bar{S}^*}{\bar{S}} \mu_H \, \mu_Z = \frac{N^*}{N} \frac{\bar{S}^*}{\bar{S}} \mu_C \tag{3.5}$$

or

$$\mu_C^* = \frac{N^*}{N} \frac{S^*}{N^*} \frac{N}{S} \mu_C = \frac{S^*}{S} \mu_C \tag{3.5$'$}$$

The claims variance becomes:

$$\sigma_C^{2*} = \mu_Z^* \left(\sigma_H^{2*} + \mu_H^{2*}\right) = \tag{3.6}$$

$$= \frac{N^*}{N} \mu_Z \left(\sigma_H^2 + \mu_H^2\right) \left(\frac{\bar{S}^*}{\bar{S}}\right)^2 =$$

$$= \frac{N^*}{N} \left(\frac{\bar{S}^*}{\bar{S}}\right)^2 \sigma_C^2$$

or

$$\sigma_C^{2*} = \frac{N^*}{N} \frac{S^{*2}}{N^{*2}} \frac{N^2}{S^2} \sigma_C^2 = \frac{S^{*2}}{S^2} \frac{N}{N^*} \sigma_C^2 \tag{3.6$'$}$$

Example:

Putting $N = 10{,}000$

$N^* = 8{,}000$

$\bar{S}^* = \dfrac{\bar{S}}{2}$

in the equations (3.5) and (3.6) we obtain:

$$E^*(C) = \mu_C^* = \frac{8{,}000}{10{,}000} \frac{\bar{S}}{2} \frac{1}{\bar{S}} \mu_C = \frac{0.8}{2} \mu_C = 0.4\, \mu_C$$

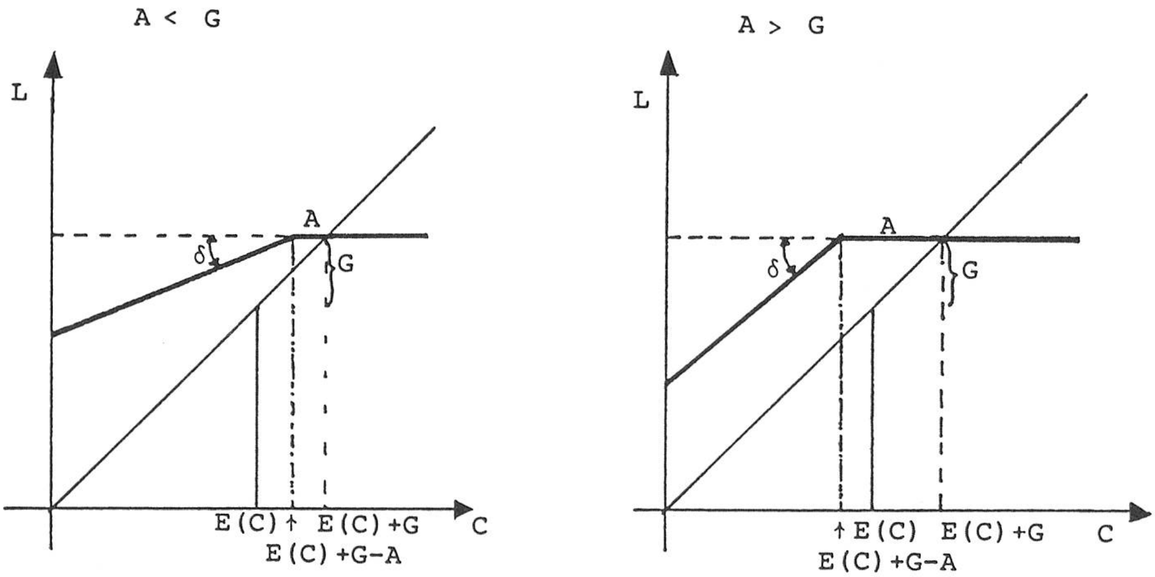
and

$$\mathrm{Var}^*(C) = \sigma_C^{2*} = \frac{8{,}000}{10{,}000} \left(\frac{\bar{S}}{2} \frac{1}{\bar{S}}\right)^2 \sigma_C^2 = 0.8 \frac{1}{4} \sigma_C^2$$

$$= 0.2 \sigma_C^2$$

4 Computation of the percentage of profit share b

Applying the results of chapters 2 and 3 we are now able to calculate the percentages of profit b for different choices of a and g, and also for different sizes of groups. Applying again the "principle of equivalence" (1.2) and taking into account the model (1.5) c),



the following equation holds:

$$E(C)=\pi-E(R)=\underbrace{E(C)+G}_{\text{max. payment}}-E(R) \tag{4.1}$$

where

$$E(R)=\int_0^{E(C)+G-A} \tan\delta\,[E(C)+G-A-c]\,f(c)dc : \text{Expected refund.} \tag{4.2}$$

[Thereby $f(c)$ is the density (2.1) obtained in chapter 2.]

(4.1), (4.2) lead to:

$$\underset{\substack{\uparrow\\(1.7)}}{b}=\tan\delta=\frac{G}{\int_0^{E(C)+G-A}[E(C)+G-A-c]\,f(c)\,dc} \tag{4.3}$$

5 Results

A computer programme has been prepared. The input data are $E(C)$ and the parameters μ and σ of the total claims density. The output gives the value of b ($\tan \delta$) for different loadings $(g-1)$ and deductions $(1-a)$.
Computations for schemes with 1,000; 2,000; ...; 10,000; 15,000 and 20,000 lives have been carried out. It is not possible to include all the results in this paper, but the following table shows an example of results for a premium loading of 5% and for schemes with 1,000; 5,000 and 10,000 lives.

$(g-1)$	$(1-a)$	b (1,000)	b (5,000)	b (10,000)
5%	4.0%	0.13345	0.24031	0.32516
	8.0%	0.14445	0.27291	0.38397
	12.0%	0.15710	0.31299	0.46044
	16.0%	0.17174	0.36295	0.56186
	20.0%	0.18883	0.42620	0.69946

Appendix 1 shows, for a fixed deduction $(1-a)$, how the percentage of profit share b varies with the profit loading $(g-1)$, whilst Appendix 2 shows how b depends on the deduction $(1-a)$ for a fixed loading $(g-1)$. In both cases the figures for schemes with 1,000; 5,000; 10,000; 15,000 livres are shown. For small schemes a linear dependence can be seen.

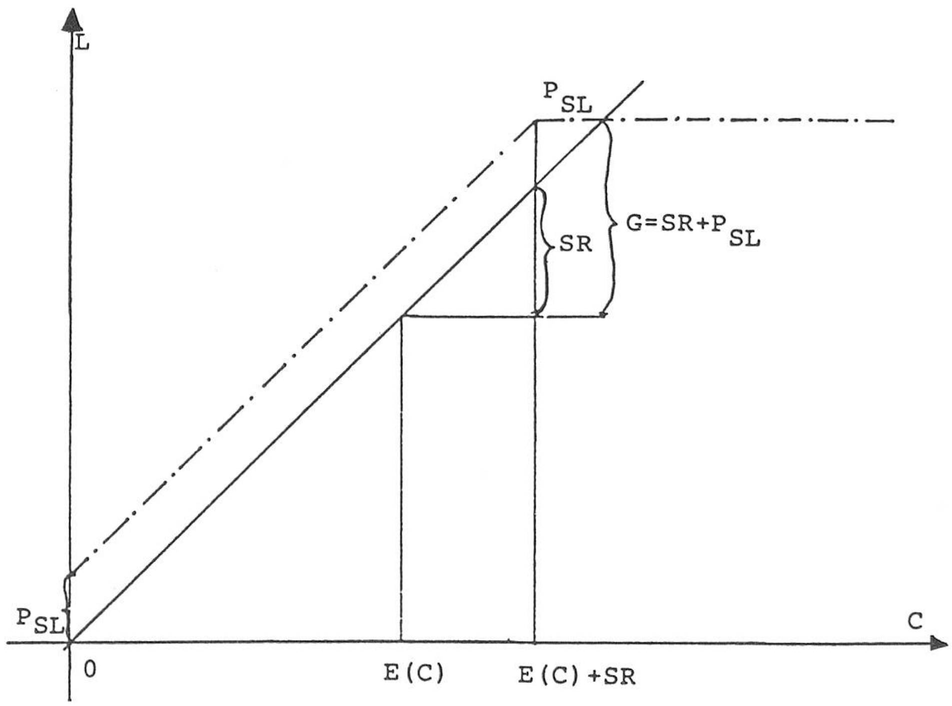
In Appendix 3 the percentage of profit share b is presented depending on the number of lives in the scheme. Only a few combinations of values of g and a are shown, but it should be enough to show that after having fixed the loading $(g-1)$ and the deduction $(1-a)$, for a number of lives between two main schemes (e.g. 6,000 and 7,000), a linear interpolation between the percentage of profit share b of the respective schemes is possible.
With the two parameters a and g, it is possible to produce many different sets of refund formulae, all of them equally acceptable from the actuarial point of view. Let us assume $a = .85$ which seems to be typical in practice. Thus, we get the following results for the refund formula: $R = b\,(.85\, g\, P - c)$.

Life-years	$(g-1)\%$	$b\%$	$(g-1)\%$	$b\%$
1,000	20	52		
2,000		65	25	75
3,000		75	20	
4,000		85	17	
5,000		93	14	
6,000	20	100	12	
10,000			8	
15,000			6	
20,000			4	75

6 Stop-Loss Reinsurance

As already pointed out in Model (1.5), the Stop-Loss Reinsurance can be considered a special case of the refund problem.

$$\text{Set: } \left.\begin{aligned} \tan\delta &:= 1 \\ G-A &:= SR \\ P_{SL} &:= A \end{aligned}\right\} \longrightarrow G = SR + P_{SL} \tag{6.1}$$



Introducing the changes suggested in (6.1) into formula (4.1), we obtain:

$$1 = \frac{SR + P_{SL}}{\int\limits_{0}^{E(C)+SR} [(E(C) + SR) - c]\, f(c)\, dc} \tag{6.2}$$

that leads to:

$$P_{SL} = \int\limits_{0}^{E(C)+SR} [(E(C) + SR - c) f(c) dc - SR. \tag{6.3}$$

Formula (6.3) is equivalent to

$$P_{SL} = \int\limits_{E(C)+SR}^{\infty} [c - (E(C) + SR)]\, f(c) dc \tag{6.4}$$

which is the typical computation of the Stop-Loss premium.

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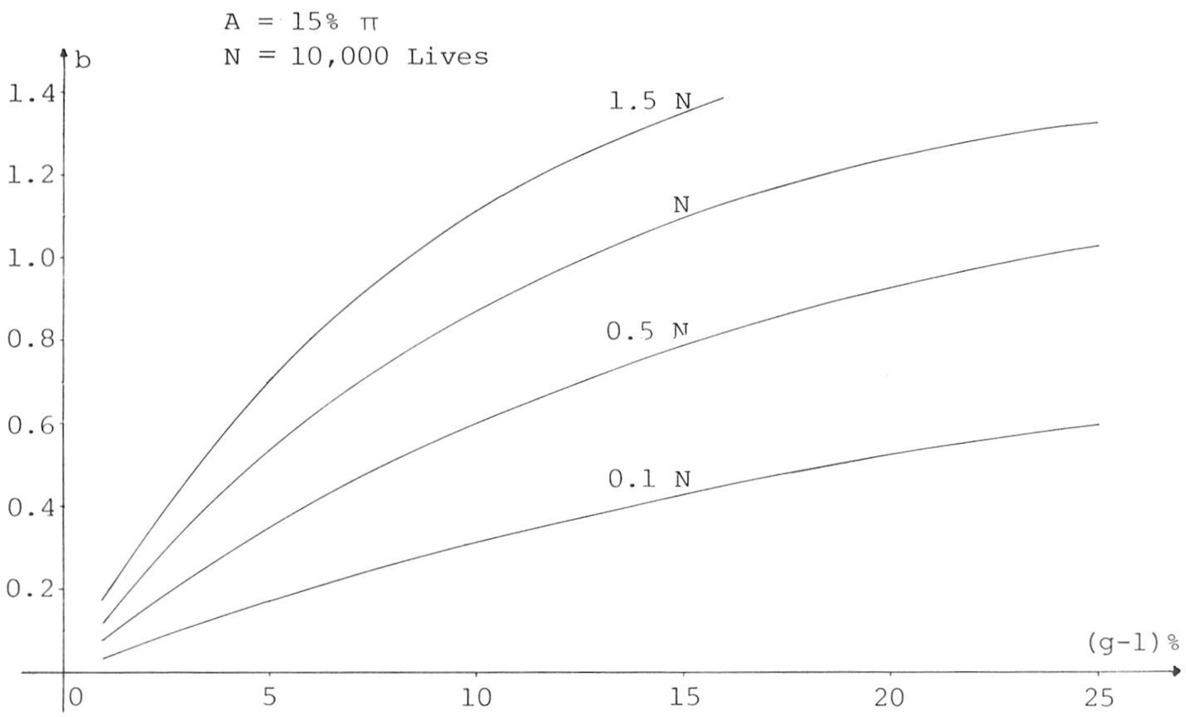
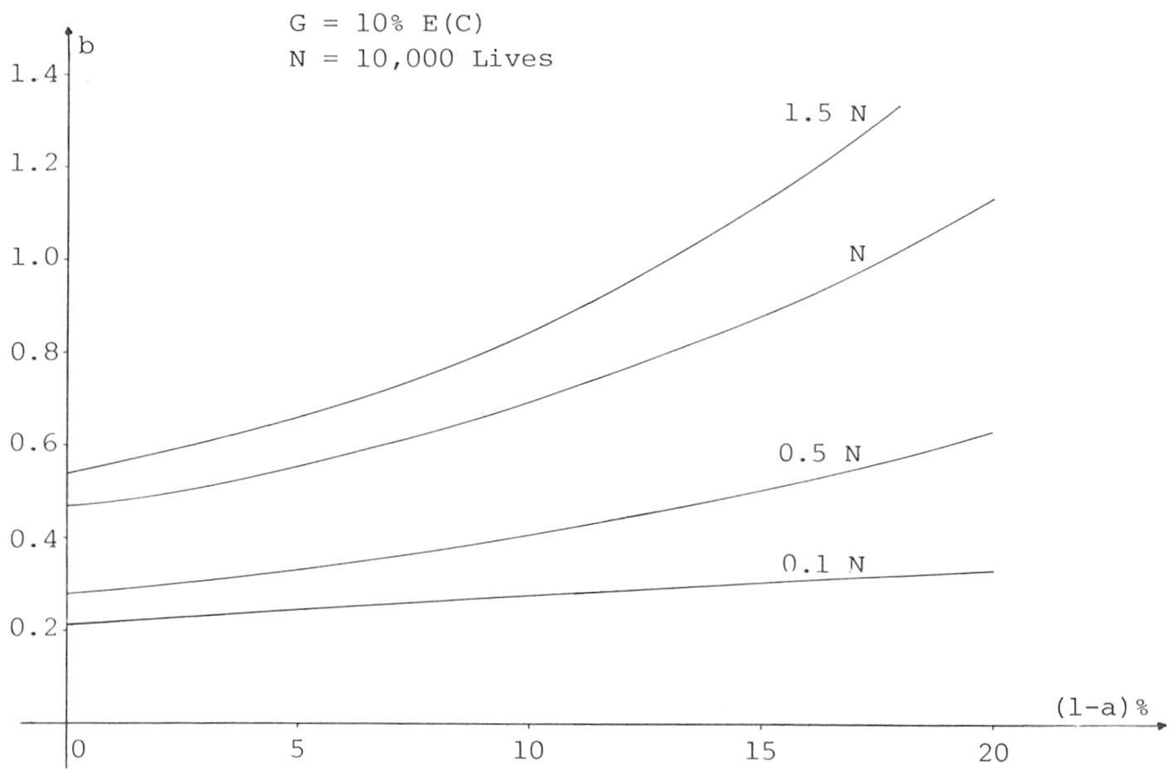
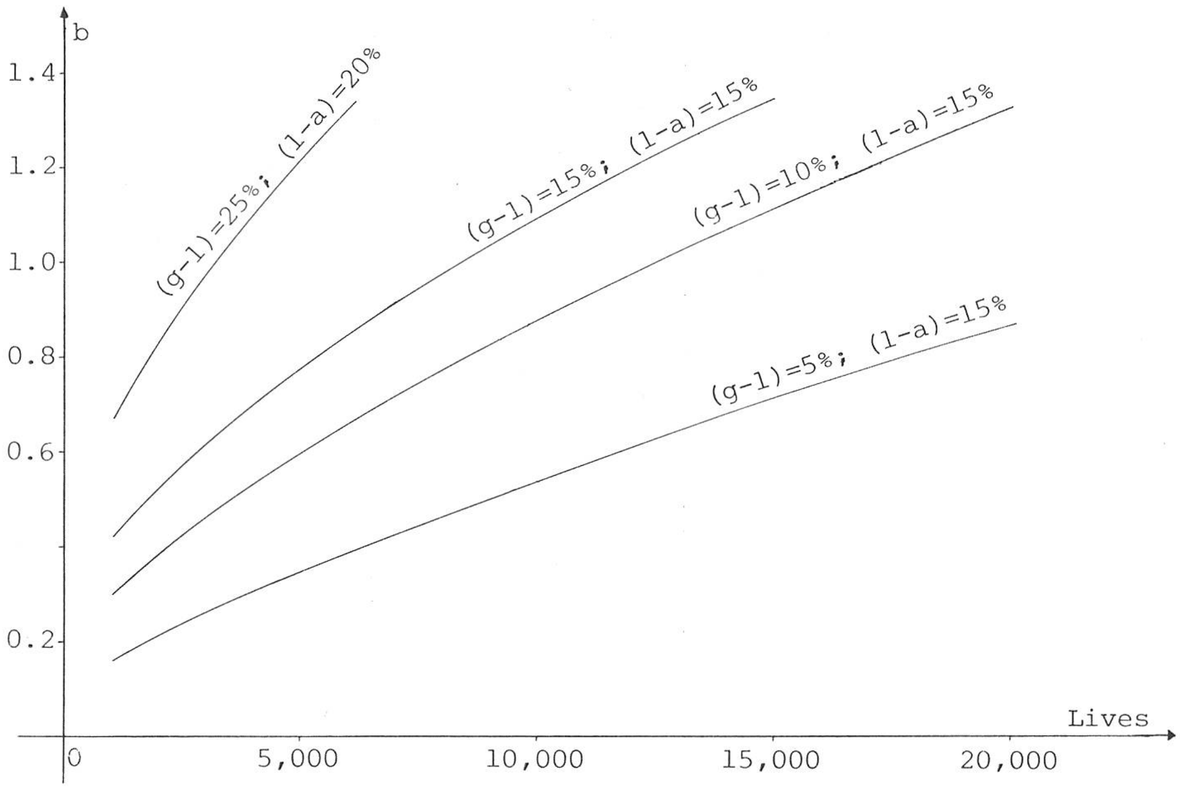
Appendix 1



Appendix 2



Appendix 3


b
1.4
1.2
1.0
0.8
0.6
0.2
(g-1)=25%; (1-a)=20%
(g-1)=15%; (1-a)=15%
(g-1)=10%; (1-a)=15%
(g-1)=5%; (1-a)=15%
Lives
0
5,000
10,000
15,000
20,000

Summary

The purpose of this paper is to establish a Refund Formula depending on the scheme size and the loading of the non-profit premium for the group life portfolio. Some practical results based on the total claim distribution of a group of 10,000 lives are shown.

Zusammenfassung

Ziel dieses Artikels ist es, eine Gewinnformel für die Kollektiv-Lebensversicherung in Abhängigkeit zur Gruppengrösse und zum Prämienzuschlag zu ermitteln. Es werden noch einige praktische Resultate, basierend auf der Totalschadenverteilung einer Gruppe mit 10000 Versicherten, gezeigt.

Résumé

L'auteur de l'article se propose de construire une formule de participation aux bénéfices pour les assurances de groupe, dépendant du nombre d'assurés et de la surprime. Il donne également quelques résultats pratiques basés sur la distribution du risque total d'un groupe de 10000 assurés.