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An Alternative Dividend Policy for an Insurance Company

1. Introduction

1.1 Dividend policies were first studied in connection with actuarial problems by *De Finetti* [5] in a critical review of the now obsolete "collective risk theory".
The center of this theory was the probability that an insurance company should remain solvent for ever, provided that no changes were made in its operating procedures. This probability will inevitably be zero, unless the company allows its reserves to grow without limit. Hence the ruin probability does not appear to be of any practical interest.

1.2 *De Finetti* argued that there must be a limit to the amount of profits which an insurance company can usefully accumulate. Beyond this limit the utility of profit *(vincite utile)* will be greater if it is paid out as dividend, than if it is added to the company's reserves. To support his argument *De Finetti* discussed a very simple example, based on a two-point distribution.
In the following Section we shall give a short presentation of *De Finetti*'s model, in a more general form. In Section 3 we shall consider an alternative model.

2. De Finetti's Model

2.1 Assume that an insurance company in successive operating periods underwrites identical portfolios, and take the following elements as given:

S = the company's initial capital

P = the premium received by the company at the beginning of each operating period

$f(x)$ = the probability density of claims paid by the company in each operating period

If S_t is the company's capital at the end of period t, and x_{t+1} the claims paid by the company during period $t+1$, the company's capital at the end of period $t+1$ will be:

$$S_{t+1} = S_t + P - x_{t+1}$$

Assume further that the company operates under the following conditions:

(i) If $S_t < 0$, the company is insolvent, or "ruined", and cannot operate in any of the following periods.

(ii) If $S_t > Z$, the company pays a dividend $s_t = S_t - Z$.

2.2 Under these conditions the company will make a series of dividend payments $s_1, s_2, \ldots, s_t, \ldots$ The elements in the series are stochastic variables, and we shall consider the expected discounted sum of these payments. We shall write

$$V(S, Z) = \sum_{t=1}^{\infty} v^t E\{s_t\}$$

for the expected discounted sum of the dividend payments from a company which has initial capital S.

From the operating conditions it follows that

$$V(S, Z) = 0 \qquad \text{for} \qquad S < 0$$

$$V(S, Z) = S - Z + V(Z, Z) \qquad \text{for} \qquad S > Z$$

It is easy to see that for $0 \leq S \leq Z$ the function $V(S, Z)$ must satisfy the integral equation

$$V(S, Z) = v \int_0^{S+P} V(S+P-x) f(x) \, dx$$

For $Z - P < S \leq Z$ the equation can be written in the form

$$V(S, Z) = v \int_Z^{S+P} \{x - Z + V(Z, Z)\} f(S+P-x) \, dx + v \int_0^Z V(x, Z) f(S+P-x) \, dx \tag{1}$$

This is an integral equation of Fredholm's type, and it is known that it has a unique continuous solution.

2.3 A problem which appears to be of obvious interest is to determine the value of Z which maximizes $V(S, Z)$. This value will represent a dividend policy which can be considered as optimal, if the company's objective is to maximize expected dividend payments. The problem has been considered by a number of authors, i.a. *Borch* [1], [2] and [3], *Bühlmann* [4], *Gerber* [6], *Hallin* [7] and *Morrill* [8].

Formally the solution to the problem is given by the value of Z which satisfies the equation

$$\frac{\partial V(S, Z)}{\partial Z} = 0 \tag{2}$$

The integral equation (1) is however difficult to handle, and a direct solution of (2) does not appear feasible. An indirect approach leads to a number of challenging mathematical problems, for instance

(i) Equation (2) may not have a unique solution, and it is not obvious that the solution is independent of S.

(ii) It is not obvious that the optimal dividend policy is of the simple form assumed by *De Finetti,* i.e. that it can be described by a single number Z.

We shall not discuss these questions, but we shall instead ask if the problem itself really is relevant.

The problem may well be as irrelevant to insurance as the ruin probability of collective risk theory.

3. An Alternative Model

3.1 The operating conditions in *De Finetti*'s model specify when a dividend shall be paid, but not when the company's equity capital shall be strengthened. Both questions should be of equal importance to management, so let us assume that an insurance company holds an equity capital Z, and consider the following policy:

If claims in a period amount to x

(i) The company pays out a dividend max $(P-x, 0)$.

(ii) An amount min $(x-P, Z)$ is paid into the company as new equity capital.

This policy implies that if the company is solvent at the end of a period, it will enter the next period with an equity capital Z.

The expected discounted value of the dividend payment at the end of the first period is

$$v \int_0^{P+Z} (P-x) f(x)\, dx$$

and the probability that the company shall be able to operate in the second period is

$$Pr\,(x \leq P+Z) = F(P+Z).$$

It then follows that the expected discounted sum of all payments is

$$W(Z) = v \int_0^{P+Z} (P-x)\, f(x)\, dx \sum_{n=0}^{\infty} \left[vF(P+Z)\right]^n$$

or

$$W(Z) = \frac{v \int_0^{P+Z} (P-x)\, f(x)\, dx}{1 - vF(P+Z)} = \frac{v\left\{\int_0^{P+Z} F(x)\, dx - ZF(Z+P)\right\}}{1 - vF(P+Z)} \tag{3}$$

3.2 Assume now that the company seeks to determine the value of Z which maximizes (3), i.e. to find the optimal amount of capital which the owners should put at risk in their company.
The first order condition, $W'(Z) = 0$, takes the form

$$Z\left\{1 - vF(P+Z)\right\} = v\left\{\int_0^{P+Z} F(x)\, dx - ZF(P+Z)\right\} \tag{4}$$

or

$$Z = v \int_0^{P+Z} F(x)\, dx \tag{5}$$

From (4) or (5) Z can be determined. Comparison of (3) and (4) shows that for the optimal Z we have

$$Z = W(Z).$$

This simple condition expresses the obvious, that the maximal capital one should put into a venture is equal to the expected present value of the return.

3.3 At first sight it may seem unrealistic to assume that the company can increase its equity capital whenever it wants. If however the insurance company is owned by a holding company, it will clearly be in the interest of the owners to see that the insurance company enters every period with the optimal equity capital.
In practice there may be difficulties for an independent insurance company which has to obtain new equity capital from financial markets. New capital will usually be needed after an unfavorable underwriting period, and this may not be quite the moment to go to the market for new capital. If however it is evident that the unfavorable result is due exclusively to random fluctuations, we must expect that the capital market will realize that it is a good investment to bring the company's capital up to the optimal level.

3.4 $F(P+Z)$ is the probability that the company shall be solvent at the end of an underwriting period. Usually the solvency condition laid down by the government will take the form

$$\alpha \leq F(P+Z) \tag{6}$$

where α is close to unity.
The optimal capital Z, determined by (5), will clearly depend on the premium P, and it is easy to show that Z will increase with P. Differentiation of (5) with respect to P gives

$$\frac{dZ}{dP} = \frac{vF(P+Z)}{1 - vF(P+Z)} > 0.$$

Hence the situation can be summed up as follows:
For given P, the insurance company will determine the optimal value of Z from (5). If this value of Z is greater than necessary to satisfy (6), the company will operate with a higher solvency margin than required by the government. If the value of Z determined by (5) is too small to satisfy (6), the company will be obliged to maintain greater reserves than it considers optimal.

3.5 It may be useful to close by considering a special case and a numerical example. If we take $f(x) = e^{-x}$, (5) takes the form

$$Z = v\{P + Z - 1 + e^{-P-Z}\}.$$

For $v = 0.9$ we find the following values:

Premium = P	Optimal reserve = Z	Ruin probability = e^{-P-Z}
0.9	0.8	0.1827
1.0	1.1	0.1225
1.1	1.5	0.0743
1.2	2.1	0.0269
1.5	4.4	0.0022

The table shows that higher premiums automatically lead to higher solvency margins. Higher premiums mean that the game becomes more favorable to the insurance company, and hence that ruin will create a greater loss. This will of course give the company an incentive to increase the probability of staying in the game.
In the example expected claims $E\{x\} = 1$. It may be surprising that also for $P < 1$ it is optimal to risk some equity capital in the insurance business.

The explanation is that the company operates with limited liability, i.e. it can under no circumstances lose more than its equity capital. Hence the game may be favorable, also when $P < E\{x\}$.

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Abstract

The paper gives a brief review of the current theory of insurance dividends. It is then assumed that at the end of an operating period, an insurance company can decide either to pay a dividend, or to obtain new equity capital. It is indicated that this assumption leads to a far simpler theory.

Zusammenfassung

Die Arbeit vermittelt zunächst einen kurzen Abriss der Dividendentheorie für Versicherungen. Es wird dann unterstellt, dass eine Versicherungsgesellschaft am Ende einer Periode die Wahl hat, entweder eine Dividende auszurichten oder ihr Kapital zu erhöhen. Es wird darauf hingewiesen, dass diese Hypothese zu einer einfacheren Theorie führt.

Résumé

L'article passe rapidement en revue la théorie usuelle des dividendes en assurance. Il part de l'hypothèse qu'à la fin d'une période la compagnie d'assurance a le choix entre le versement d'un dividende et une augmentation de capital. L'article indique que cette hypothèse conduit à une théorie nettement plus simple.