

# Loss-reserving by kernel regression

Autor(en): **Kremer, Erhard**

Objektyp: **Article**

Zeitschrift: **Mitteilungen / Schweizerische Vereinigung der  
Versicherungsmathematiker = Bulletin / Association Suisse des  
Actuaires = Bulletin / Swiss Association of Actuaries**

Band (Jahr): - **(1989)**

Heft 1

PDF erstellt am: **13.07.2024**

Persistenter Link: <https://doi.org/10.5169/seals-967209>

## **Nutzungsbedingungen**

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern. Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden. Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

## **Haftungsausschluss**

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

---

 ERHARD KREMER, Hamburg

## Loss-Reserving by Kernel Regression

### 1 Introduction

In the following one of the main problems of nonlife insurance mathematics is reconsidered, i.e. how to calculate loss reserves. In principal this problem reduces to the prediction of the future development of yet unsettled claims. In the last few years a vast body of papers appeared in actuarial journals treating the problem of loss reserving in nonlife insurance, worth mentioning are e.g. the articles of *Bühlmann et al.* (1980), *De Jong / Zehnwirth* (1983), *De Vylder* (1978), *Kremer* (1984), *Linnemann* (1980), *Straub* (1971), *Taylor* (1977), *Verbeek* (1972) and the actuarial surveys of *Van Eeghen* (1981) and *Taylor* (1986). The papers contain many different approaches, some authors use so-called separation techniques (see *Linnemann* (1980), *Taylor* (1977), *Verbeek* (1971)), some others apply credibility methods (see *De Vylder* (1982), *Straub* (1971)), recently also time series methods were adopted (see *De Jong and Zehnwirth* (1983), *Lemaire* (1981) and *Kremer* (1984)). Some methods were tested and applied on empirical data, see e.g. *Taylor* (1981), *Pater* (1987), and found to be adequate for practical determination of loss reserves. Though at the present state there exist already quite many different methods, the author thought again about that topic and recognized an interesting connection between loss reserving and nonparametric kernel-regression estimation, a topic discussed extensively in journals on Mathematical Statistics during the past twenty-five years (see *Devroye* (1981), *Gleblicki* (1984), *Nadaroya* (1964)). This noticed correspondence led the author to write this further paper, presenting a new loss reserving or better predicting approach, based on modified nonparametric estimation methods.

### 2 The Loss Reserving Problem

Let  $Y_{ij}$ ,  $j = 1, \dots, m$ ,  $i = 1, \dots, m$  be (nonnegative) random variables on a probability space  $(\Omega, \mathcal{A}, P)$ ,  $Y_{ij}$  denoting the claims number or the claims size (per claim) of a collective of risks in the development year  $j$  and in respect of the accident year  $i$ . Known is only the triangle

$$Y_{\nabla} = (Y_{ij}, j = 1, \dots, m - i + 1, i = 1, \dots, m),$$

representing the past claims development. The problem consists in calculating or estimating the unknown loss reserves for the accident years  $i = 2, \dots, m$ . For this one has to calculate or estimate the future growth

$$R_i(Y_{\nabla}) = Y_{im} - Y_{i,m-i+1}, \quad (2.1)$$

for each accident year  $i = 2, \dots, m$ . In case of claims numbers one speaks of the IBNR-reserving problem, in case of claim sizes of the IBNER-reserving problem. Here IBNR or IBNER are abbreviations for 'incurred but not reported' or 'incurred but not enough reserved'. Replacing in case of claim numbers  $Y_{ij}$  by  $N_{ij}$ , in case of claim sizes  $Y_{ij}$  by  $S_{ij}$ , the *loss reserve* for the IBNR-claims of accident year  $i$  is given by

$$R_i(N_{\nabla}) \cdot S_{im} \quad (2.2)$$

and the loss reserve for the IBNER-claims of the accident year  $i$  by

$$R_i(S_{\nabla}) \cdot N_{i,m-i+1}. \quad (2.3)$$

The total loss reserve for the accident year no.  $i$  then is given by the sum of the reserves for the IBNR-claims and the IBNER-claims.

Obviously claims reserving, in the above sense, reduces to predicting the unknown  $Y_{ij}$ ,  $j = m - i + 2, \dots, m$ ,  $i = 2, \dots, m$  from the known triangle  $Y_{\nabla}$ . Fortunately concepts and ideas of the Mathematical Statistics can be adapted to this situation, more concretely methods of the estimation and prediction theory (see e.g. *Lehmann* (1983) and *Granger/Newbold* (1977)). As already mentioned in the introduction many adequate actuarial methods, which are modifications of corresponding mathematical-statistical methods, do yet exist and are successfully applied in the insurance practice. In the sequel a further new one is presented.

### 3 The Optimal Predictions

Denote by  $L_2$  the set of all square-integrable random variables  $X$ , defined on a fixed probability space  $(\Omega, \mathcal{A}, P)$ . By identifying  $X$  with the equivalence class of all  $\tilde{X}$  with

$$\tilde{X} = X \quad \text{almost surely,}$$

the  $L_2$  becomes a Hilbert space with scalar product and norm respectively

$$\begin{aligned}\langle X, Y \rangle &= E(X \cdot Y), \\ \|X\|_2 &= (E(X^2))^{1/2}\end{aligned}$$

for  $X, Y$  of the  $L_2$ . We denote by  $M_{\nabla}$  the set of all  $Z$  of the  $L_2$ , depending measurably on  $Y_{\nabla}$ , this means that there is a measurable function  $g$  on the  $m \cdot (m + 1)/2$ -dimensional real space with

$$Z = g(Y_{ij}, j = 1, \dots, m - i + 1, i = 1, \dots, m).$$

$M_{\nabla}$  is the class of all predictors from the triangle  $Y_{\nabla}$ . It is obvious to define the *optimal forecast* (or prediction) of  $Y_{ij}$  from the triangle  $Y_{\nabla}$  as the unique element  $\widehat{Y}_{ij} \in M_{\nabla}$  satisfying

$$\|Y_{ij} - \widehat{Y}_{ij}\|_2 \leq \|Y_{ij} - Z\|_2$$

for all  $Z \in L_2$ , i.e. as the (orthogonal) projection of  $Y_{ij}$  on the closed linear subspace  $M_{\nabla}$  of the  $L_2$ . As wellknown this projection can be represented as a conditional expectation operator, more concretely

$$\begin{aligned}\widehat{Y}_{ij} &= E(Y_{ij} | Y_{\nabla}) \\ &= E(Y_{ij} | Y_{kl}, l = 1, \dots, m - k + 1, k = 1, \dots, m)\end{aligned}\tag{3.1}$$

(compare e.g. Theorem 2.15 in *Kremer (1985)*). On the additional assumption that the row vectors

$$(Y_{i1}, \dots, Y_{im}), \quad i = 1, \dots, m\tag{3.2}$$

are stochastically independent,

one has the more simple formula for the optimal prediction:

$$\widehat{Y}_{ij} = E(Y_{ij} | Y_{il}, l = 1, \dots, m - i + 1).\tag{3.3}$$

According to the above written, the loss reserving problem reduces simply to the determination of the conditional expectations given in (3.1) or (3.3) respectively.

#### 4 Estimating the Optimal Predictions

Without additional assumptions no simple explicit formula can be given for the conditional expectations of (3.1) and (3.3). In a former article the author took a nonstationary, autoregressive model for the development of the  $Y_{ij}$ ,  $j = 1, 2, \dots, m$ , i.e.

$$Y_{ij} = \sum_{l=1}^p a_{jl} \cdot Y_{i,j-l} + b_{ij} + e_{ij}, \quad i = 1, \dots, m \quad j = 1, \dots, m \quad (4.1)$$

with real parameters  $a_{jl}$ ,  $b_{ij}$  and random error terms  $e_{ij}$  (see *Kremer (1984)*) and gave in his Theorem 1 simple recursions for the conditional expectation or optimal prediction (3.3). As variants of the classical least squares estimation practicable estimation methods were given in his Theorem 2 for the unknown parameters  $a_{jl}$ ,  $b_{jl}$  (see also *Pater (1987)*). Instead of assuming a parametric model, e.g. something like (4.1), let us only assume that for a given  $p \geq 1$ :

$$\left. \begin{aligned} &Y_{ij} \text{ depends only through the} \\ &Y_{i,m-i-p+2}, \dots, Y_{i,m-i+1} \\ &\text{from the } Y_{i1}, \dots, Y_{i,m-i+1} \\ &\text{for } j = m - i + 2, \dots, m \text{ and } i = 2, 3, \dots, m. \end{aligned} \right\} \quad (4.2)$$

Then (3.3) simply reduces to the predictor

$$\hat{Y}_{ij} = E(Y_{ij} | Y_{i,m-i-p+2}, \dots, Y_{i,m-i+1}) \quad (4.3)$$

for  $j = m - i + 2, \dots, m$  and  $i = 2, \dots, m - p + 1$ .

Now, how to estimate this slightly more simple conditional expectation without any additional parametric assumption?

For this one can use ideas of a special field of nonparametric estimation theory, the so-called *nonparametric regression estimation*. For adapting we assume in addition to (3.2) and (4.2) that

$$\left. \begin{aligned} &\text{one has given (random or nonrandom) variables } A_i, B_i, \\ &i = 1, 2, \dots, m \text{ such, that for the transformed variables} \\ &X_{ij} = \frac{(Y_{ij} - B_i)}{A_i} \\ &\text{the random vectors} \\ &(X_{i1}, \dots, X_{im}), \quad i = 1, 2, \dots, m \\ &\text{are identically distributed.} \end{aligned} \right\} \quad (4.4)$$

These transformations represent possible trend effects in the different accident years, which have to be eliminated in advance. Besides this, in case of claim sizes one has to inflation adjust all claims in advance, leading to the above random variables  $Y_{ij}$ . Assuming for the  $X_{ij}$  in place of the  $Y_{ij}$  the conditions (3.2) and (4.2) (which for nonrandom  $A_i, B_i$  clearly carry over), obviously the optimal predictions of the unknown  $X_{ij}$  from the triangle

$$X_{\nabla} = (X_{ij}, j = 1, \dots, m - i + 1, i = 1, \dots, m)$$

are

$$\widehat{X}_{ij} = E(X_{ij} | X_{i, m-i-p+2}, \dots, X_{i, m-i+1}) \quad (4.5)$$

for  $j = m - i + 2, \dots, m$  and  $i = 2, \dots, m - p + 1$ .

If the  $A_i, B_i$  are stochastically independent of the  $X_{ij}$ ,  $j = m - i - p + 2, \dots, m - i + 1$  (which in case of nonrandom  $A_i, B_i$  clearly is satisfied), the predictions  $\widehat{Y}_{ij}$  of (4.3) obviously can be computed from predictions  $\widehat{X}_{ij}$  of (4.5) according

$$\widehat{Y}_{ij} = A_i \cdot \widehat{X}_{ij} + B_i. \quad (4.6)$$

Consequently it remains to give a formula or an approximate formula for the predictions (4.5) on the assumptions (3.2), (4.2) and (4.4) with the  $X_{ij}$  instead of the  $Y_{ij}$ . In order to get a good approximation procedure, we extend the above setting a little bit. We assume that we have some more complete claims developments of past years indexed by  $i = -n, -n + 1, \dots, 0$ . This means we have in addition the random variables

$$Y_{i1}, \dots, Y_{im}, \quad i = -n, -n + 1, \dots, 0$$

with the claims amount or claims number  $Y_{ij}$  of the  $j$ -th development year with respect to the  $i$ -th accident year. We assume (4.4) for the whole set of claims data, with accident year index running through the values  $i = -n, \dots, m$ . Finally (3.2) is supposed for the whole set of transformed data

$$(X_{i1}, \dots, X_{im}), \quad i = -n, \dots, m$$

(clearly for nonrandom  $A_i, B_i$  this follows from the same statement for the original data  $Y_{ij}$ ).

In this setting approximate formulas for (4.5) can be given by the use of *kernel regression estimators* of order  $p$ . We take a nonnegative function  $K$  on the

$p$ -dimensional real space of row vectors into the real line such, that there exist constants  $c_1, c_2$  and  $r > 0$  with

$$c_1 \leq K(x) \leq c_2,$$

holding on the circle  $\{x: \|x\| \leq r\}$  of the  $p$ -dimensional space of row vectors. Here and in the following  $\|\cdot\|$  denotes the euclidean norm. Furthermore choose a sequence  $(h_n)_{n \geq 1}$  with

$$h_n \rightarrow 0, \quad \text{for } n \rightarrow \infty.$$

With this notation define for  $i = 2, \dots, m-p+1$  and  $j = m-i+2, \dots, m$  the following functions  $\mu_{ij}$  on the  $p$ -dimensional real space of row vectors  $x = (x_1, \dots, x_p)$  according

$$\mu_{ij}(x) = \frac{\sum_{l=-n}^{m-j+1} K\left(\left(\frac{x_1 - X_{l,m-i-p+2}}{h_{m-j+n+2}}\right), \dots, \left(\frac{x_p - X_{l,m-i+1}}{h_{m-j+n+2}}\right)\right) \cdot X_{lj}}{\sum_{l=-n}^{m-j+1} K\left(\left(\frac{x_1 - X_{l,m-i-p+2}}{h_{m-j+n+2}}\right), \dots, \left(\frac{x_p - X_{l,m-i+1}}{h_{m-j+n+2}}\right)\right)}.$$

The sense of this becomes clear in the following general Theorem, giving the fundamental property of these functions  $\mu_{ij}(x)$ .

*Theorem*

Assume for the sequence  $(h_n)_{n=1,2,\dots}$  that one has

$$n \cdot \frac{h_n^p}{\log(n)} \rightarrow \infty, \quad \text{for } n \rightarrow \infty.$$

and that (what is satisfied in insurance):

$$|X_{ij}| \leq \bar{x} < \infty, \quad \text{for all } i \text{ and } j.$$

Then one has

$$\lim_{n \rightarrow \infty} (\mu_{ij}(x)) = E(X_{ij} | X_{i,m-i-p+2} = x_1, \dots, X_{i,m-i+1} = x_p)$$

in almost all  $x = (x_1, \dots, x_p)$ . □

*Proof*

We reformulate the statement in a more general way: Let  $(X, Y), (X_1, Y_1), \dots, (X_n, Y_n)$  be independent, identically distributed (row) random vectors in the corresponding  $(p + 1)$ -dimensional real space with  $Y, Y_i \in L_2$ . Similar to the definition of  $\mu_{ij}$  we define  $m_n$  on the real space of  $p$ -dimensional row vectors according:

$$m_n(x) = \sum_{i=1}^n W_{ni}(x) \cdot Y_i$$

for  $x = (x_1, \dots, x_p)$  with the weights

$$W_{ni}(x) = \frac{K\left((x_1 - X_{i1})/h, \dots, (x_p - X_{ip})/h\right)}{\sum_{i=1}^n K\left((x_1 - X_{i1})/h, \dots, (x_p - X_{ip})/h\right)}$$

for  $(X_{i1}, \dots, X_{ip}) = X_i$  and  $K, (h_n)_{n \geq 1}$  declared as above. Obviously this is a generalized version of the above setup of predicting the  $X_{ij}$  from the  $X_{il}, l = m - i - p + 2, \dots, m - i + 1$ . The statement of the Theorem simply reduces to

$$|m_n(x) - m(x)| \rightarrow 0 \quad \text{a.e. for } n \rightarrow \infty, \quad (4.7)$$

for almost all  $x$ , with the definition:

$$m(x) = E(Y | X_{i1} = x_1, \dots, X_{ip} = x_p).$$

This statement is nothing else but the Theorem 4.2 in Devroye (1981). For sake of completeness the main steps of Devroye's proof are given in a very short style. One has obviously:

$$\begin{aligned} |m_n(x) - m(x)| &\leq \left| \sum_{i=1}^n W_{ni}(x) \cdot (Y_i - m(X_i)) \right| + \\ &\quad + \sum_{i=1}^n W_{ni}(x) \cdot |m(X_i) - m(x)|. \end{aligned} \quad (4.8)$$

For given  $\varepsilon > 0$  one can give constants  $c_1, c_2$  such that

$$\begin{aligned} P\left(\left| \sum_{i=1}^n W_{ni}(x) \cdot (Y_i - m(X_i)) \right| > \varepsilon \mid X_1, X_2, \dots, X_n\right) \\ \leq c_1 \cdot \exp\left(-c_2 \cdot \sup_i W_{ni}(x)\right) \end{aligned} \quad (4.9)$$



and the second term in (4.8) is bounded from above by

$$U_n(x) = \left( \frac{c_2}{c_1} \right) \cdot \sum_{i=1}^n |m(X_i) - m(x)| \cdot \left( \frac{1_{A_{in}(x)}}{\sum_{i=1}^n 1_{A_{in}(x)}} \right),$$

where  $1_{A_{in}(x)}$  is the indicator function of the event

$$A_{in}(x) = \{ \|X_i - x\| \leq r \cdot h_n \}.$$

One concludes that for given  $\varepsilon > 0$  there exist constants  $c_3, c_4$  such, that

$$P\left(|U_n(x) - E(U_n(x))| > \varepsilon\right) \leq c_3 \cdot E\left(\exp(-c_4 \cdot N_n(x))\right),$$

where  $N_n(x)$  is distributed like  $1_{A_{in}(x)}$ , i.e. is binomially distributed with parameters  $n$  and  $p_n(x)$ , satisfying:

$$n \cdot \frac{p_n(x)}{\log(n)} \rightarrow \infty \quad \text{for } n \rightarrow \infty,$$

for almost all  $x$ . One can show that for almost all  $x$ :

$$\sum_{n=1}^{\infty} E\left(\exp(-s \cdot N_n(x))\right) < \infty$$

for all  $s > 0$ , implying with Borel-Cantelli-Lemma that for almost all  $x$

$$U_n(x) - E(U_n(x)) \rightarrow 0 \quad \text{a.e., for } n \rightarrow \infty.$$

Since

$$E(U_n(x)) \rightarrow 0 \quad \text{for } n \rightarrow \infty$$

in almost all  $x$ , one has that the second term on the right hand side of (4.8) converges a.e. for almost all  $x$  to zero. Also the right hand side of (4.9) can be bounded with a suitable  $c_5$  by

$$c_1 \cdot \exp(-c_5 \cdot N_n(x)),$$

implying again with the Borel-Cantelli Theorem and Lebesgues Theorem, that also the first term on the right hand side of (4.8) converges a.e. to zero for almost all  $x$ . This completes the proof of the statement (4.7).  $\square$

According to this result the  $\widehat{X}_{ij}$ ,  $i = 2, \dots, m - p + 1$ ,  $j = m - i + 2, \dots, m$ , calculated by

$$\widehat{X}_{ij} = \mu_{ij}(X_{i,m-i-p+2}, \dots, X_{i,m-i+1})$$

can be used as *approximations* to the *optimal prediction*  $\widehat{X}_{ij}$ . With the modification of (4.6)

$$\widehat{Y}_{ij} = A_i \cdot \widehat{X}_{ij} + B_i$$

one has the desired *loss predicting procedure* of forecasting  $Y_{ij}$  by  $\widehat{Y}_{ij}$ .

In the application of this loss predicting procedure the following things have to be considered:

1. In the case  $p > 1$  there is a terminating problem for the accident years  $i > m - p + 1$ . For these years one can use functions  $K$  defined on lower dimensioned spaces and proceed like above.
2. The question appears, how to choose the sequence  $(h_n)_{n \geq 1}$ . A criterion for the choice of the  $h_n$ ,  $n \geq 1$  is given in the above Theorem, i.e. choose the sequence such, that  $h_n \rightarrow 0$  and

$$n \cdot \frac{h_n^p}{\log(n)} \rightarrow \infty, \quad n \rightarrow \infty.$$

3. A lot of freedom in the above general method lies in the choice of the function  $K$ , called *Kernel-function*, and the appropriate dimension  $p$  of the definition space. In the case  $p = 1$  the author got good results with a kernel of the type:

$$\begin{aligned} K(x) &= |x|^{-1}, & \text{for } x \notin (-\varepsilon, \varepsilon) \\ &= \bar{x}, & \text{for } x \in (-\varepsilon, \varepsilon) \end{aligned}$$

where  $\bar{x}$  is a comparably large and  $\varepsilon$  a comparably small positive value. When applying the method, one should try to find an adequate kernel function on some given test data.

For illustration of the above method a simple example is cited.

## 5 An Example

In the above notation with  $m = 3$ ,  $n = 0$  we take the truncated triangle of the claims sizes

$$Y_{0j}, \quad j = 1, 2, \dots, m$$

$$Y_{ij}, \quad j = 1, \dots, m - i + 1, \quad i = 1, 2, \dots, m.$$

given by:

	$j = 1$	2	3	4
$i = 0$	23.2	33.8	37.3	38.9
1	25.8	37.3	42.9	45.6
2	22.1	30.3	30.7	
3	35.9	43.0		
4	34.9			

Obviously the rows seem to be not identically distributed. We have to transform the data like in (4.4). We choose simply  $A_i = Y_{i1}$ ,  $B_i = 0$ , i.e. we take:

$$X_{ij} = \frac{Y_{ij}}{Y_{i1}}, \quad \text{for all } i \text{ and } j,$$

and use a kernel function with  $p = 1$ , i.e. calculate the approximate prediction according:

$$\hat{X}_{ij} = \frac{\sum_{l=0}^{m-j+1} K\left(\frac{(X_{lj} - X_{l,m-i+1})}{h_{m-j+2}}\right) \cdot X_{lj}}{\sum_{l=0}^{m-j+1} K\left(\frac{(X_{lj} - X_{l,m-i+1})}{h_{m-j+2}}\right)}$$

for  $j = m - i + 2, \dots, m$ ,  $i = 2, \dots, m$ . According to the remark 2. one can take e.g.

$$h_n = \left(\frac{1}{n}\right)^{1/2}$$

and according to the remark 3.:

$$K(x) = |x|^{-1}, \quad \text{for } x \notin (-\varepsilon, \varepsilon)$$

$$= 1000, \quad \text{for } x \in (-\varepsilon, \varepsilon)$$

with  $\varepsilon = 1000^{-1}$ . This implies the completed rectangle of the  $X_{ij}$ ,  $i = 0, \dots, m$ ,  $j = 1, \dots, m$ :

	$j = 1$	2	3	4
$i = 0$	1.0000	1.4569	1.6078	1.6767
1	1.0000	1.4457	1.6628	<u>1.7674</u>
2	1.0000	1.3710	<u>1.3891</u>	1.7170
3	1.0000	<u>1.1978</u>	1.5316	1.7230
4	<u>1.0000</u>	1.3678	1.5532	1.7220

Multiplication of the rows with the corresponding  $Y_{i1}$ -values yields the completed lower part of the rectangle of the  $Y_{ij}$ -values:

		37.95
	54.98	61.86
47.74	54.21	60.10

These values are basis for giving the loss reserves of section 2. □

Prof. Dr. E. Kremer  
Verein zur Förderung der  
Angewandten Mathematischen  
Statistik und Risikotheorie e.V.  
Robert-Koch-Strasse 14a  
D-2000 Hamburg 20

## References

- Bühlmann, H./Schnieper, R./Straub, E.* (1980): Claims reserves in casualty insurance based on a probabilistic model. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, 21–45.
- De Jong, P./Zehnwirth, B.* (1983): A random coefficient approach to claims reserving. *Journal of the Institute of Actuaries*, 157–181.
- Devroye, L.* (1981): On the almost everywhere convergence of nonparametric regression function estimates. *Annals of Statistics*, 1310–1319.
- De Vylder, F.* (1978): Estimation of IBNR claims by least squares. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, 249–254.
- Gleblicki, W./Krzyszak, A./Pawlak, M.* (1984): Distribution-free pointwise consistency of kernel regression estimate. *Annals of Statistics*, 1570–1575.
- Granger, C. W. J./Newbold, P.* (1977): *Forecasting Economic Time Series*. Academic Press, San Francisco.
- Kremer, E.* (1984): A class of autoregressive models for predicting the final claims amount. *Insurance: Mathematics and Economics*, 111–119.
- Kremer, E.* (1985): *An Introduction to Actuarial Mathematics* (in German). Vandenhoeck & Ruprecht, Göttingen/Zürich.
- Lehmann, E.* (1983): *Theory of point estimation*. John Wiley and Sons, New York.
- Lemaire, J./Melard, G./Vandermeulen, E.* (1981): Claims reserves: An autoregressive model. *Journal of forecasting*.
- Linnemann, P. U. K.* (1980): A multiplicative model of loss reserves: A stochastic process approach. *Laboratory of Actuarial Mathematics*.
- Nadaroya, E. A.* (1964): On estimating regression. *Theory of Probability and Applications*, 141–142.
- Pater, R.* (1987): The run-off-triangle: Least Squares – against chain ladder. *ASTIN Colloquium at Scheveningen*.
- Straub, E.* (1971): On the calculation of IBNR-reserves. *The price winning papers in the Boleslaw monic fund competition 1971*.
- Taylor, G. C.* (1977): Separation of inflation and other effects from the distribution of nonlife insurance claim delays. *ASTIN Bulletin*, 219–230.
- Taylor, G. C.* (1981): A comparison of methods of estimation of third party motor outstanding claims. *Lecture on the May 1981 Colloque International of the Institute des Hautes Etudes de Belgique, Brussel*.
- Taylor, G. C.* (1986): *Claims reserves in non-life insurance*. North-Holland, New York.
- Van Eeghen* (1981): *Loss reserving methods*. Survey of Actuarial Studies No. 1. Nationale Nederlanden, Research Department.
- Verbeek, H. C.* (1972): An approach to the analysis of claims experience in motor liability excess of loss reinsurance. *ASTIN Bulletin*, 195–202.

## **Summary**

A new approach to loss reserving is presented in this paper. The statistical concept of nonparametric regression is adapted to the problem of calculating IBNR or IBNER reserves. General prediction formulas for forecasting the unknown future claims development are defined with a given kernel function. The application of the resulting methods is demonstrated in an example.

## **Zusammenfassung**

In der vorliegenden Arbeit wird ein neuer Zugang für die Schätzung von Schadenreserven vorgestellt. Das statistische Konzept der nichtparametrischen Regression wird angewandt auf das Problem der Berechnung von IBNR- und IBNER-Reserven. Allgemeine Formeln zur Voraussage der unbekanntes künftigen Schadenentwicklung werden mit einer gegebenen Kernfunktion definiert. Die Anwendung der Methode wird an einem Beispiel erläutert.

## **Résumé**

Une nouvelle approche pour l'évaluation des réserves est présentée dans cet article. Le concept statistique de régression non-paramétrique est adapté au problème du calcul des réserves IBNR et IBNER. Des formules générales pour la prévision de l'évolution future et inconnue des sinistres sont définies avec une fonction noyau donnée. L'application des méthodes résultantes est illustrée par un exemple.

