Insurance premiums, the insurance market and the need for reinsurance

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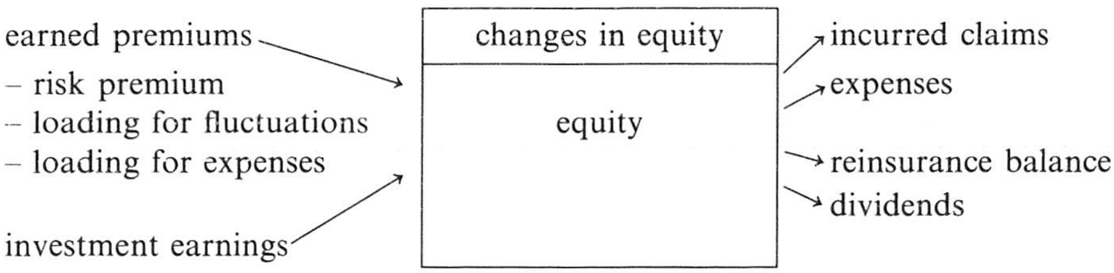
1 The Model

An insurance company can be viewed as a reservoir with an incoming and an outgoing flow of money, the difference between the two flows being accumulated into the equity of the company. (The following general model is borrowed from *Beard/Pentikäinen/Pesonen* [1]). The incoming flow consists of the premiums with their different components (risk premium, loading for fluctuations and loading for expenses) and of the net investment earnings. The outgoing flow consists of incurred claims, expenses, reinsurance balance (premium for outwards reinsurance less reinsurance recoveries) and dividends.



We make the following simplifying assumptions:

1. The expenses and the loading for expenses are identical and therefore cancel out.
2. The risk premium and the reserve for outstanding losses are both discounted. To be more precise, the earned premium of the risk year will flow in over a given period of time, the incurred losses of the risk year will be paid out over another possibly much longer period of time. We make the assumption that the risk premium is computed in such a way that the expected present value per end of the risk year of both cash flows are identical, moreover we assume that the reserve for outstanding losses is the present value per end of the risk year of future

claims settlements. The interest rate which is used for discounting is the rate of the riskless security.

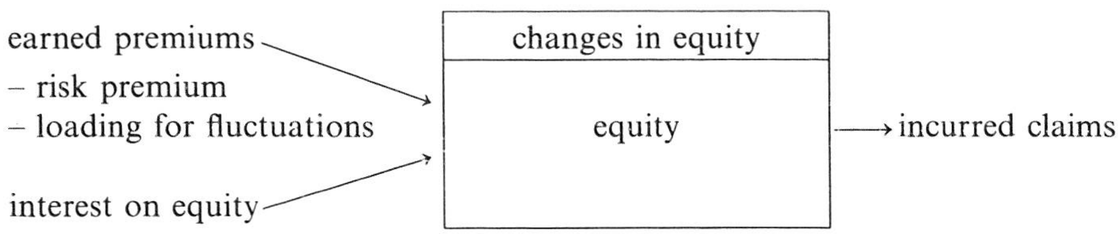
3. All the assets of the company are invested in the riskless security.

   As a consequence of assumption 2 and 3 the interest on unearned premium reserves and outstanding loss reserves does not have to be taken into account. The only interest which has to be taken into account is the interest on equity. Assumption 3 is made for didactical reasons: we want to concentrate on the fluctuations stemming from the insurance business and do not want to take into account the fluctuations in the value of the assets of the company.

4. We look at the results of the company before dividends to shareholders are paid out. Dividends and retained profit are lumped together and appear in the changes in equity. We shall slightly modify this assumption in section 2.

5. Earned premiums and incurred claims are net of premiums for outwards reinsurance and of reinsurance recoveries respectively.

Our assumptions lead to the following simplified model:



We use the following notation:

S total claims amount incurred in the risk year (net of reinsurance recoveries).

$pr[S]$ premiums earned in the risk year (net of premiums for outwards reinsurance).

ℓ loading for fluctuations

u equity of the company at the beginning of the risk year

i yield of the riskless security

Du change in equity in the risk year.

The following relations hold true:

$$pr\,[S] = E[S] + \ell$$
$$Du = pr\,[S] - S + iu$$

hence

$$E[Du] = \ell + iu = (j + i)u$$

where

$$j = \frac{\ell}{u}$$

is the additional expected rate of return to which the shareholders of the insurance company are entitled for investing a part of their assets in the insurance company. This additional rate of return is determined by the capital market, by what investors perceive as a fair compensation for assuming the insurance risk. The loading for fluctuations will therefore be determined if we can determine the amount of surplus necessary to the operation of the insurance company.

2 Required Surplus and Risk Theoretical Premium

2.1 Premium to Surplus Ratio

The premium to surplus ratio is often considered to be a good measure of the financial strength of an insurance company. Hence the required surplus is:

$$u = a \cdot E[S]$$

and the premium is:

$$pr\,[S] = E[S] + j \cdot u = E[S](1 + j \cdot a)\,.$$

In the United States a ratio of up to 2.5 to 1 is considered to be acceptable, whereas in Europe this ratio may be as high as about 5 to 1. Of course the premium which is taken into account is not the pure risk premium but the gross premium which also includes the loading for expenses and the loading for fluctuations. Assuming a 30 % expense loading and a premium to surplus ratio of 2.5 to 1 one obtains a $\simeq$ 0.5, together with an additional rate of return

j of 10 % this gives a loading for fluctuations which is approximately equal to 5 % of the pure risk premium.

2.2 α-Stability (Solvency)

The premium to surplus ratio does not take into account the variability of the total claims amount and therefore it is not a good measure of the stability of the insurance company. A more appropriate way to look at the stability of the company is the following: the amount of surplus should be such that the loss of a sizeable part of the surplus (say α) at the end of the risk period should only happen with a small probability (say ε). If $\alpha = 1$ then ε is the probability of being insolvent at the end of the risk period. Thus:

$$\text{Prob}\ [S - pr\,[S] - iu > \alpha u] = \varepsilon$$

We assume that the rate of return of the risk free asset is equal to the rate of inflation. Therefore a portion of the surplus equal to $i \cdot u$ will eroded by inflation at the end of the risk period. If we want to correct our criterion for inflation we must therefore restate it in the following way:

$$\text{Prob}\ [S - pr\,[S] > \alpha u] = \varepsilon$$

Since on the other hand:

$$pr\,[S] = E[S] + ju$$

we obtain:

$$\text{Prob}\left[\frac{S - E[S]}{\sigma[S]} > (\alpha + j)\frac{u}{\sigma[S]}\right] = \varepsilon$$

where $\sigma[S]$ is the standard deviation of the total claims amount.

Let F denote the distribution function of the standardized total claims amount:

$$F(x) = P\left[\frac{S - E[S]}{\sigma[S]} \leq x\right]$$

the required surplus is

$$u = \frac{1}{\alpha + j} F^{-1}(1 - \varepsilon)\sigma\,[S]$$

and the premium is

$$pr\,[S] = E[S] + \frac{j}{\alpha + j} F^{-1}(1 - \varepsilon)\sigma\,[S]\,.$$

Assuming

$$j = 0.10, \quad \alpha = 0.3, \quad \varepsilon = 0.01, \quad F(\cdot) \text{ standard normal distribution}$$

we obtain

$$u = 5.816\,\sigma\,[S]\,.$$
$$pr\,[S] = E[S] + 0.582\,\sigma\,[S]\,.$$

2.3 *Ruin Probability*

We assume that the portfolio is stationary and that the rate of return of the riskless security is equal to the inflation rate. Therefore inflation and interest earned on equity cancel out. The surplus at the beginning of the first risk year is u. In each risk year a dividend $j_0 \cdot u$ is paid to shareholders. The premium after payment of this dividend is:

$$pr\,[S] = E[S] + j' \cdot u$$

where

$$j' = j - j_0$$

The initial surplus is chosen in such a way that the ruin probability is ε. We now derive an approximation for this initial surplus.
Let R be the adjustment coefficient:

$$E\left[e^{-R(pr\,[S] - S)}\right] = 1$$

using a martingale argument it is easy to show that

$$\varepsilon \leq e^{-Ru}$$

which is cramer's inequality. Replacing $\leq$ by $\simeq$ in the above equation one obtains

$$R \simeq \frac{|\log \varepsilon|}{u}.$$

On the other hand, from the very definition of R it follows that:

$$pr\,[S] = \frac{1}{R} \log E[e^{RS}].$$

Developing the right hand side of the above equation into a Taylor series one obtains:

$$pr\,[S] \simeq E[S] + \frac{1}{2}\ \text{VAR}\ [S] \cdot R = E[S] + \frac{1}{2}\ \text{VAR}\ [S] \frac{|\log \varepsilon|}{u}$$

on the other hand

$$pr\,[S] = E[S] + j'u$$

from which one obtains the required surplus to operate the insurance company at ruin probability ε:

$$u = \left(\frac{|\log \varepsilon|}{2j'}\right)^{\frac{1}{2}} \sigma\,[S]$$

and the premium is

$$pr\,[S] = E[S] + j \cdot \left(\frac{|\log \varepsilon|}{2 \cdot j'}\right)^{\frac{1}{2}} \sigma\,[S]$$

In the special case where $j = j'$, i.e. where no dividends are paid to shareholders we obtain the same surplus as and half the loading of *Bühlmann* [2] who derived those quantities from a different model.

Assuming

$$j = 0.10, \quad j' = 0.08, \quad \varepsilon = 0.01$$

we obtain

$$u = 5.365\,\sigma\,(S)$$

$$pr\,[S] = E[S] + 0.536\,\sigma\,[S]\,.$$

2.4 *Remarks*

In actuarial literature a distinction is sometimes made between a loading for fluctuations and a loading for profit. In this paper they are identical. It is felt that a split between the two loadings is arbitrary. Management will tend to view any premium component in addition to the pure risk premium and the loading for expenses as a loading for profit whereas ratemakers will view it as a loading for fluctuations.

If we determine the amount of required surplus based on α-stability or on the probability of ruin we obtain a premium for the whole portfolio which is computed according to the standard deviation principle. The premium to surplus ratio leads to the expected value principle. From now on we shall focus on α-stability and on the probability of ruin and therefore restrict ourselves to the standard deviation principle for the premium of the portfolio as a whole. Since α-stability is a criterion which applies to one risk year at a time, it can easily cope with growing portfolios and there is no need to explicitly differentiate between dividends to shareholders and retained profit. It is therefore a more flexible criterion than ruin probability. In the case of a stationary portfolio, both criteria lead to very similar results.

The assumption that the rate of return of the riskless security is equal to the inflation rate can be dropped at the price of a clumsier notation. All the results remain valid.

3 The Market premium and the Need for Reinsurance

At this stage we briefly summarize what we have done. We have assumed that the premium for the whole portfolio is of the following form:

$$Pr\,[S] = E[S] + ju$$

where the additional rate of return on equity j is determined by the capital market and where the amount of equity required to assume the insurance business is derived from some risk theoretical considerations. By using two different stability criteria we have arrived at a required surplus of the following form:

$$u = k \cdot \sigma[S].$$

Where k is computed according to 2.2 or to 2.3.
Therefore, in order for the company to be able to do business at the required level of stability, the loading for the whole portfolio must satisfy the following constraint:

$$\ell \geq j \cdot k \cdot \sigma[S]$$

On the other hand, the loading by which the company can increase the pure risk premium of a given individual risk is limited by the insurance market. We assume that the market premium for a given risk X is of the following form:

$$\Pi[X] = E[X] \cdot (1 + \lambda)$$

Where the loading factor λ is different for different lines of business but is the same for all risks in a given line of business.
Assuming that the whole portfolio of the company consists of risks from a single line of business (the problem of the combination of different lines of business will be addressed later) we obtain the following market premium for the portfolio as a whole

$$\Pi[S] = E[S] \cdot (1 + \lambda).$$

The loading for the whole portfolio must therefore satisfy the following market constraint:

$$\ell \leq \lambda \cdot E[S].$$

The market constraint and the risk theoretical constraint can only by satisfied simultaneously if:

$$j \cdot k \cdot \sigma[S] \leq \lambda E[S] \tag{3.1}$$

or:

$$\frac{\sigma[S]}{E[S]} \leq \frac{\lambda}{j \cdot k} \tag{3.2}$$

The condition states that the variation coefficient of the portfolio as a whole cannot exceed some upper bound. If this is not the case, the variation coefficient must be reduced. This can be achieved through portfolio growth (at least for uncorrelated risks) or through reinsurance. As far as reinsurance is concerned we shall distinguish between proportional and non-proportional reinsurance. For a definition of the different types of reinsurance and reinsurance treaties we refer to *Straub* [3].
In the case of proportional reinsurance (e.g. quota share or surplus treaties), we shall assume that the loading for fluctuations of the reinsurer is of the same form as the original market loading. If we denote the net and the reinsured portfolio by S_n and S_r respectively (3.1) becomes:

$$j \cdot k \cdot \sigma[S_n] \leq \lambda \cdot E[S] - \lambda' \cdot E[S_r] = \lambda \cdot E[S_n] + (\lambda - \lambda') \cdot E[S_r]$$

where λ' is the loading factor of the reinsurer.
If in addition we assume:

$$\lambda' = \lambda$$

we obtain:

$$j \cdot k \cdot \sigma[S_n] \leq \lambda \cdot E[S_n]$$

and therefore:

$$\frac{\sigma[S_n]}{E[S_n]} \leq \frac{\lambda}{j \cdot k} \tag{3.3}$$

from which the retention of a surplus treaty can be derived. An example will be given below.
It is interesting to note that since a quota share treaty does not affect the variation coefficient of the portfolio, there is no need for a quota share treaty in our model. It must however be remembered that a basic assumption of our model is that the company can always raise the required surplus $u = k \cdot \sigma[S]$ provided that it is able to produce an expected annual profit $j \cdot u$. If this

assumption is not satisfied, then the company will have to cede a quota share of its business.
It is also interesting to note that the required surplus has decreased from

$$u = k \cdot \sigma[S]$$

to

$$u = k \cdot \sigma[S_n]$$

and that the (risk) premium to surplus ratio is:

$$\frac{E[S_n]}{u} = \frac{j}{\lambda}$$

In the case of non-proportional reinsurance (e.g. excess of loss or stop loss treaties) which is subject to far bigger fluctuations than proportional reinsurance, it is not realistic to assume that the security loading of the reinsurer is of the same form as the original loading. In general it is considerably larger than the original loading and depends on the retention of the ceding company. We shall assume that the reinsurance loading is proportional to the variance of the ceded risk; condition (3.1) then becomes:

$$j \cdot k \cdot \sigma[S_n] \leq \lambda E[S] - \lambda_r \cdot \sigma^2[S_r] \tag{3.4}$$

when λ_r denotes the loading factor applied by the reinsurer to the variance of the ceded risk. The priority of the non proportional cover can be derived from this equation. An example will be given below.

Example 1: Determining the critical size of a portfolio

For a portfolio of independent and homogenous risks (e.g. a motor third party liability portfolio), condition (3.2) becomes:

$$\frac{v}{\sqrt{n}} \leq \frac{\lambda}{j \cdot k}$$

where v denotes the variation coefficient of the loss distribution of an individual risk and n is the number of risks. The size above which the portfolio does not need any reinsurance cover is therefore:

$$n = \left(\frac{j \cdot k \cdot v}{\lambda}\right)^2.$$

With the same choice of parameters as in section 2.3 and with

$$v = 7 \quad \text{and} \quad \lambda = 0.01$$

we obtain

$$n = 141\,038$$

Example 2: Determining the line (retention) of a surplus treaty

Under the assumption that the distribution of the total claims amount is the compound Poisson distribution with expected number of claim n, we have

$$E[S_n] = n \cdot E[X_n]$$
$$\sigma^2 [S_n] = n \cdot E[X_n^2]$$

where X_n denotes the individual net claims. We denote by

SI the sum insured of an individual risk,

D the claims degree (i.e. the ratio of an individual claim to the sum insured of the corresponding risk)

and by

$G(s) =$ Prob $[SI \leq s]$ the distribution of sums insured,

$H(d) =$ Prob $[D \leq d]$ the distribution of claims degree.

We make the additional assumption that the sums insured and the claims degree are independent random variables. In the case of a surplus with line m, the distribution of net individual claims is:

$$F(x) = \text{Prob } [X_n \leq x]$$
$$= \int_0^m H\left(\frac{x}{s}\right) \cdot dG(s) + H\left(\frac{x}{m}\right)(1 - G(m)).$$

From which it follows

$$E[X_n^k] = \int_0^\infty x^k \int_0^m h\left(\frac{x}{s}\right) dG(s) \frac{dx}{s} + \int_0^\infty x^k h\left(\frac{x}{m}\right)(1 - G(m)) \frac{dx}{m}$$
$$= E[D^k]\left(\int_0^m s^k \cdot dG(s) + m^k \cdot (1 - G(m))\right).$$

The idea to rewrite the k-the moment of X_n in the above way is borrowed from *Straub* [3]. Condition (3.3) now becomes

$$\frac{\left(\int_0^m s^2 \cdot dG(s) + m^2 \cdot (1 - G(m))\right)^{\frac{1}{2}}}{\int_0^m s \cdot dG(s) + m \cdot (1 - G(m))} \leq \frac{\lambda}{j \cdot k} \frac{E[D]}{(E[D^2])^{\frac{1}{2}}} \sqrt{n}$$

It is easily seen that the left hand side of the above expression is an increasing function of m, its limit is equal to one when m tends to zero and equal to

$$\frac{(E[SI^2])^{\frac{1}{2}}}{E[SI]}$$

when m tends to infinity. Therefore, if the right hand side is large enough, there exists a value of m for which both sides of the above expression are equal; this is the line of the surplus treaty.

We shall now illustrate this result with a numerical example. We consider a fire portfolio with $n = 100\,000$ risks. The sums insured are distributed according to a lognormal distribution with $\mu = 0.4340$ and $\sigma = 1.1529$, which means that the average sum insured is 3 and the standard deviation is 5. We think of the currency unit as being a Million Swiss Francs. The probability of a claim on a given risk in a given year is $p = 0.01$; given that there is a claim, the claims degree is distributed according to a Beta-distribution with parameters 0.1 and 0.9, therefore:

$$E[D] = 0.1 \cdot p = 0.001$$
$$(E[D^2])^{\frac{1}{2}} = 0.235\sqrt{p} = 0.0235.$$

Assuming that j and k are as in section 2.3 and that the market loading factor λ is equal to 0.05, the right hand side of the above inequality is equal to 1.25. To find the value of m for which the left hand side is equal to 1.25 is an exercise in numerical integration. In the present case the line m is equal to 4.5.

The average gross sum insured was SFr. 3 millions, the average net sum insured is SFr. 2.03 millions. The gross pure risk premium was SFr. 300 millions, the net pure risk premium is SFr. 203 millions, the expected gross profit was SFr. 15 millions, the expected net profit is SFr. 10.15 millions. The surplus necessary to operate the insurance company (assuming that our fire portfolio is the only business underwritten by the company) is SFr. 101.5 millions.

Example 3: Determining the retention of an excess of loss treaty.

In the case of an excess of loss treaty with retention r, the individual claims are split between the insurer and the reinsurer in the following way:

$$X_n = X \wedge r = \begin{cases} x & x \leq r \\ r & x > r \end{cases}$$

$$X_r = (X - r)^+ = \begin{cases} 0 & x \leq r \\ x - r & x > r \end{cases}$$

where X_n and X_r denote the net and the reinsured individual claims respectively. Under the assumption that S has a compound Poisson distribution we have

$$\begin{aligned} E[S] &= n \cdot E[X] \\ \sigma^2 [S_n] &= n \cdot E[(X \wedge r)^2] \\ \sigma^2 [S_r] &= n \cdot E[(X - r)^{+2}] \end{aligned}$$

and condition (3.4) now reads

$$j \cdot k \cdot n^{\frac{1}{2}} \left(E[(X \wedge r)^2] \right)^{\frac{1}{2}} \leq \lambda \cdot n \cdot E[X] - \lambda_r \cdot n \cdot E[(X - r)^{+2}]$$

From which the retention can be determined if we know the distribution of individual claims.

We illustrate this fact with a numerical example. We assume that the fire risks of the preceding example are also covered against elemental perils. We consider the set of those covers as a separate portfolio. Since the different risks are strongly correlated we no longer look at individual risks but at the portfolio as a whole. We assume that the total claims amount for a given risk year has a compound Poisson distribution with an expected number of loss events equal to 2 and with the total loss stemming from one event being distributed according to a Pareto distribution with shape parameter equal to 1, floor parameter 1 and truncated at 100:

$$P[X \leq x] = \begin{cases} 0 & x \leq 1 \\ 1 - \frac{1}{x} & 1 < x \leq 100 \\ 1 & x > 100 \end{cases}$$

The currency unit is a million Swiss Francs. The gross pure risk premium is equal to SFr. 9.210 millions and we assume that the market loading factor λ is equal to 0.50 giving an expected profit of 4.605. The loading factor for elemental perils is much lager than the loading factor in pure fire insurance, since due to the strong correlation of individual risks, the fluctuations of the results are much larger too. The loading factor applied by the reinsurer to the variance of the assumed risk is $\lambda_r = 0.01$ per million Swiss Francs. The values of j and k are as in section 2.3. Thus the above inequality becomes

$$1.073\,(r - 1 - \ell n(r))^{\frac{1}{2}} \leq 4.605 - 0.04\,r\left(\frac{100}{r} - 1 - \ell n\left(\frac{100}{r}\right)\right).$$

The maximum retention for which the inequality is satisfied is $r =$ SFr. 3.6 millions. With this retention, the expected net profit is SFr. 1.23 millions and the amount of surplus necessary to assume the net risk is SFr. 12.3 millions.

Example 4: Combining uncorrelated portfolios

Let us assume that we have two uncorrelated subportfolios with total claims amount ${}^1S$ and ${}^2S$ respectively. We shall suppose that ${}^1S$ is protected by proportional reinsurance and ${}^2S$ by non-proportional reinsurance in such a way that the ruin probability of each subportfolio is ε (or alternatively that the probability of losing a share α of the surplus allocated to each subportfolio is ε) We then have

$$j \cdot k(\varepsilon) \cdot \sigma\,({}^1S_{n_\varepsilon}) = \lambda_1 E[{}^1S_{n_\varepsilon}]$$
$$j \cdot k(\varepsilon) \cdot \sigma\,({}^2S_{n_\varepsilon}) = \lambda_2 E[{}^2S] - \lambda_r \sigma^2\,[{}^2S_r]$$

where $k(\varepsilon)$ is computed according to 2.3 (or alternatively to 2.2). λ_1 and λ_2 denote the market premium loadings of the first and second subportfolio respectively and ${}^iS_{n_\varepsilon}$ $(i = 1, 2)$ are the net total claims amounts. By $\Lambda_1(\varepsilon)$ and $\Lambda_2(\varepsilon)$ we denote the right hand side of the first and second equation respectively. $\Lambda_i(\varepsilon)$ is the net expected profit of subportfolio i.

If we now pool the two uncorrelated subportfolios, it is intuitively obvious that the overall ruin probability will be smaller than ε. In order to operate at an overall ruin probability ε we therefore choose $\varepsilon' > \varepsilon$ in such a way that

$$j \cdot k(\varepsilon) \cdot \left(\sigma^2({}^1S_{n_{\varepsilon'}}) + \sigma^2({}^2S_{n_{\varepsilon'}})\right)^{\frac{1}{2}} = \Lambda_1(\varepsilon') + \Lambda_2(\varepsilon').$$

A generalisation of the above procedure to n subportfolios is straightforward. We illustrate this result with a numerical example. In the case of example 2 and 3 we had respectively

$$\Lambda_1(0.01) = 10.15$$
$$\sigma({}^1S_{n_{0.01}}) = 18.92$$

and

$$\Lambda_2(0.01) = 1.23$$
$$\sigma({}^2S_{n_{0.01}}) = 2.29.$$

The standard deviation of the combined portfolio is:

$$\left(\sigma^2({}^1S_n) + \sigma^2({}^2S_n)\right)^{\frac{1}{2}} = 19.06$$

and the expected profit which is necessary to retribute the surplus required to assume the risk is

$$j \cdot k \cdot \left(\sigma^2({}^1S_n) + \sigma^2({}^2S_n)\right)^{\frac{1}{2}} = 10.22$$

which is less than the actual expected profit

$$\Lambda_1 + \Lambda_2 = 11.38.$$

We can therefore either run the business with more surplus than required, i.e. with a probability of ruin which is lower than 1 %, or we can increase the ruin probability of the individual subportfolios until the above equation is satisfied i.e. until the overall ruin probability of the whole portfolio is equal to 1 %. By plugging the relation

$$\sigma({}^iS_{n_{\varepsilon'}}) = \frac{\Lambda_i(\varepsilon')}{j \cdot k(\varepsilon')}$$

into the above equation we obtain

$$\frac{k(\varepsilon)}{k(\varepsilon')}\left(\Lambda_1^2(\varepsilon')+\Lambda_2^2(\varepsilon')\right)^{\frac{1}{2}}=\Lambda_1(\varepsilon')+\Lambda_2(\varepsilon')$$

from which ε' can be easily determined. For a given ε', the $\Lambda_i(\varepsilon')$ are computed as in example 2 and 3 respectively. If the left hand side of the equation is smaller than the right hand side, ε' is increased; else ε' is decreased. The process is iterated until the equation is satisfied.

In the present case we obtain $\varepsilon' = 0.025$, i.e. the ruin probability of the subportfolios is 2.5 %. The line of the surplus treaty is SFr. 8.1 millions giving a net pure risk premium of SFr. 245 millions and a net expected profit of SFr. 12.25 millions. The retention of the excess of loss treaty is 4.3 millions and the expected net profit is 1.32 millions.

The ruin probability of the combined portfolio is 1 %, as required, and the expected net profit is 13.57 millions. The amount of equity necessary to run the business has been increased to SFr. 135.7 millions.

Example 5: Determining the commitment per risk.

An interesting special case arises when individual risks are uncorrelated and of such a size that they all exceed the company's capacity. A typical example of such a situation is industrial fire insurance. It is assumed that the company computes the premium of individual risks according to the variance principle.

$$pr\,[X_i] = E[X_i] + \frac{a}{2}\,\text{VAR}\,[X_i]$$

because the risks are uncorrelated, the premium of the whole portfolio is then

$$pr\,[S] = E[S] + \frac{a}{2}\,\text{VAR}\,[S]$$

on the other hand the total premium income must be

$$pr\,[S] = E[S] + j \cdot u$$

from which a is determined

$$a = \frac{2 \cdot j \cdot u}{\text{VAR}\,[S]}\,.$$

or expressing the variance of the portfolio as a function of the equity of the company

$$a = \frac{2 \cdot j \cdot k^2}{u}$$

where k is defined according to 2.3 (or alternatively according to 2.2).
Let us now assume that the market premium for risk i is

$$\Pi[X_i] = E[X_i](1 + \lambda_i)$$

i.e. we allow the market loading to depend on the individual risk.
Let s_i denote the share of the risk i underwritten by the company. It must satisfy the following relationship

$$pr\,[s_i X_i] = E[s_i X_i] + \frac{a}{2}\,\text{VAR}\,[s_i X_i] = s_i\,\Pi[X_i]$$

from which

$$s_i = \frac{\lambda_i \cdot E[X_i]}{\text{VAR}\,[X_i]} \cdot \frac{a}{2} = \frac{\lambda_i \cdot E[X_i]}{\text{VAR}\,[X_i]}\,\frac{u}{j \cdot k^2}$$

In the special case where X is a fire risk with only total losses and with policy limit ℓ, i.e.

$$X = \begin{cases} \ell & p \\ o & 1-p \end{cases}$$

we have

$$E[X] = \ell \cdot p$$
$$\text{VAR}\,[X] = \ell^2 \cdot p(1-p) \simeq \ell^2 \cdot p \qquad \text{for} \quad p \ll 1$$

and the commitment of the company is

$$s \cdot \ell \simeq \frac{\lambda u}{j \cdot k^2}$$

With $\lambda = 0.05$ and j and k in section 2.3, we obtain

$$s \cdot \ell \simeq 0.017\,u$$

i.e. for such a risk the commitment of the company is approximately equal to 2 % of its equity.

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Literature

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Summary

A simple model of an insurance company is presented. From this model we derive the risk theoretically required premium for a given portfolio. The insurance market usually does not allow the company to charge the full required premium. The risk theoretical constraint and the market constraint can be reconciled by reinsuring the portfolio. The retention of the ceding company is derived as a by-product.

Zusammenfassung

Ausgehend von einem einfachen Modell einer Versicherungsgesellschaft, wird die risikotheoretisch notwendige Prämie für ein gegebenes Portefeuille ermittelt. Aufgrund der Marktverhältnisse kann diese Prämie gewöhnlich nicht voll einverlangt werden. Es wird gezeigt, dass die risikotheoretischen Erfordernisse und die Einschränkungen des Marktes durch Rückversicherung des Portefeuilles in Einklang gebracht werden können. Der Selbstbehalt des Erstversicherers wird dabei als Nebenergebnis hergeleitet.

Résumé

Un modèle simple d'une compagnie d'assurance est présenté. De ce modèle nous dérivons la prime requise pour un portefeuille donné. En général la prime de marché est inférieure à la prime requise par la théorie du risque. Les deux conditions peuvent être réconciliées en réassurant le portefeuille. La rétention de la compagnie découle du modèle.