A probabilistic model for calculation of a single premium for short credit life insurance with included inflation

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1 Introduction

A credit life insurance is usually defined as a term life insurance purchased in conjunction with consumer credit transaction, which provides a death benefit sufficient to pay off the credit obligation in the event of the death during the term of coverage. The most common form of credit life insurance is gross coverage which means that the amount of insurance covers the principal outstanding (remaining of debt).

In practice a standard decreasing term insurance is commonly used to guarantee repayment of a loan, provided that the debt outstanding also decreases "almost" linearly under the amortization plan of the loan. A combination between a standard decreasing term insurance and appropriate level term usually gives a sufficient approximation for coverage. This method is applicable when the interest rate or inflation rate is reasonably low.

In my country, we have a special situation. Due to historic reasons and relatively high inflation, our banks regularly adjust monthly installments on the basis of the inflation rate. If the inflation rate at the end of the month is equal to j (for example 1 %), the following month the installment is multiplied by $(1 + j)$. This process is repeated until the debt is repaid. It can easily be noticed that the process is finite and that the number of installments remains the same.

Example: Let assume that the monthly inflation rate is constant during the year and is equal to 3 % per month. As an example we take a loan of 10 000 units for 1 year duration with 15 % annual interest rate. The starting annuity is equal to 898,14 payable at the end of the month. The amortization plan of the loan looks as follows:

Period in months	Interests paid	Installment	Principal outstanding
k	I_k	N_k	P_k
0	0,00	0,00	10000,00
1	117,15	898,14	9495,58
2	111,24	925,09	8942,18
3	104,76	952,84	8336,92
4	97,67	981,43	7676,75
5	89,93	1010,87	6958,49
6	81,52	1041,19	6178,78
7	72,38	1072,43	5334,10
8	62,49	1104,60	4420,74
9	51,79	1137,74	3434,83
10	40,24	1171,87	2372,29
11	27,79	1207,03	1228,85
12	14,40	1243,24	0,00

In this case we calculate a starting annuity from

$$N_1 = 1000/a_1^{(12)}, \qquad a_1^{(12)} = \frac{1-w}{o^{(12)}/12},$$

where

$$w = \frac{1}{1.15} \quad \text{and} \quad o^{(12)} = 12[1.15^{1/12} - 1].$$

We denote the discount factor by w and the nominal interest rate equivalent to $o = 15\,\%$ by $o^{(12)}$.
By the definition we get N_{k+1} recursively

$$N_{k+1} = 1.03 N_k, \qquad k = 2, \dots, 11.$$

It is also obvious that the principal outstanding and the paid interest may be written as

$$I_k = \frac{o^{(12)}}{12} P_{k-1},$$

$$P_k = 1.03\left(1 + \frac{o^{(12)}}{12}\right) P_{k-1} - 1.03 N_k, \qquad k = 1, \dots 12.$$

In general case, where j is an inflation rate and o is a credit rate, both with the same conversion period, we find that

$$P_k = (1+j)[(1+o)P_{k-1} - N_k]\,, \qquad k = 1, \ldots, n\,.$$

2 Definitions

We will follow international notation, which can be found for example in [1]. Let denote the future lifetime of a person x by T. We define $K = [T]$, the number of completed future years of a life aged x and S, the fraction of a year during which person is alive in the year of death. By the definition we have

$$T = K + S\,.$$

A discrete random variable K has probability distribution given by

$$P[K = k] = {}_kp_x q_{x+k}$$

and the random variable S has a continuous distribution between 0 and 1. We will assume that K and S are independent random variables. We will also define the random variable with discrete uniform distribution

$$S^{(m)} = \frac{1}{m}[mS + 1]\,,$$

where m is a positive integer. Clearly K and $S^{(m)}$ are also independent variables. In addition we denote the present value of claim amount in time T from beginning of the issue of policy by Z.
Let i be the technical rate of interest for premium calculation with discount factor $v = 1/(1+i)$ and force of interest $\delta = \ln(1+i)$. Accordingly we denote the interest rate of the credit as o with $w = 1/(1+o)$ and $\sigma = \ln(1+o)$. Finally we will assume the constant yearly inflation j with force of interest $\lambda = \ln(1+j)$. This assumption is reasonable for a short period (from 1 to 4 years) and relatively stable economy.

3 The Model

Let us consider a debt of size 1, which should be repaid in n years with initial monthly installment N_1. At the time point t, $0 \leq t \leq n$, the remaining debt is equal to

$$(1+j)^t \frac{a_{n-t}}{a_n} .$$

Our task is to calculate a single premium for term insurance, where the sum insured is equal to the remaining debt. The remaining debt is a discrete function of time and it is constant between two periods.

According to the notations we gave, we can write the present value of future claims as

$$Z = \begin{cases} (1+j)^{(K+S^{(m)}-1/m)} \frac{1-w^{(n+1/m-K-S^{(m)})}}{1-w^n} v^{K+S} , & \\ & K+S<n \\ 0 , & K+S \geq n \end{cases} .$$

To get the single premium we have to calculate expected value $A = E(Z)$. Following idea from [1], page 26, we divide Z into two parts

$$\begin{aligned} Z_1 &= (1+j)^{K+1-1/m} v^{K+1} (1+j)^{S^{(m)}-1} (1+i)^{1-S} , \\ Z_2 &= (1+j)^{K+1-1/m} v^{K+1} w^{n+1/m-K-1} (1+j)^{S^{(m)}-1} \\ &\quad \times (1+i)^{1-S} w^{1-S^{(m)}} . \end{aligned}$$

Then

$$E(Z) = \frac{1}{1-w^n} (E(Z_1) - E(Z_2)) .$$

In computing the net single premium we use independence of the random variables K and S. To determine $E(Z_1)$. Let's try to calculate

$E((1+j)^{S^{(m)}-1}(1+j)^{1-S})$. We get

$$F_1 = E((1+j)^{S^{(m)}-1}(1+i)^{1-S})$$
$$= \sum_{k=1}^{m} \int_{\frac{k-1}{m}}^{\frac{k}{m}} (1+j)^{k/m-1}(1+i)^{1-s}\, ds$$
$$= \frac{1}{\delta}\sum_{k=1}^{m}(1+j)^{k/m-1}(1+i)^{1-k/m}[(1+i)^{1/m}-1]$$
$$= \frac{(i-j)i^{(m)}}{\delta(1+j)m\left[\left(\frac{1+i}{1+j}\right)^{1/m}-1\right]}.$$

In a similar way we can find, that

$$F_2 = E((1+j)^{S^{(m)}-1}w^{1-S^{(m)}}(1+j)^{1-S})$$
$$= \sum_{k=1}^{m} \int_{\frac{k-1}{m}}^{\frac{k}{m}} (1+j)^{k/m-1}w^{1-k/m}(1+i)^{1-s}\, ds$$
$$= \frac{1}{\delta}\sum_{k=1}^{m}(1+j)^{k/m-1}(1+o)^{k/m-1}(1+i)^{1-k/m}[(1+i)^{1/m}-1]$$
$$= \frac{(i-j-o-jo)i^{(m)}}{\delta(1+j)(1+o)m\left[\left(\frac{1+i}{(1+j)(1+o)}\right)^{1/m}-1\right]}.$$

Hence we obtain

$$A = \frac{1}{1-w^n}\Bigg[F_1\sum_{k=0}^{n-1}(1+j)^{k+1-1/m}v^{k+1}{}_kp_x q_{x+k}$$
$$- F_2\sum_{k=0}^{n-1}(1+j)^{k+1-1/m}w^{n+1/m-k-1}v^{k+1}{}_kp_x q_{x+k}\Bigg].$$

Letting $m \to \infty$ (continuous model) we have some simplifications of upper results

$$\overline{A} = \frac{1}{1-w^n}\frac{i-j}{(\delta-\lambda)(1+j)}\sum_{k=0}^{n-1}(1+j)^{k+1}v^{k+1}{}_kp_xq_{x+k}$$
$$-\frac{1}{1-w^n}\frac{i-j-o-jo}{(\delta-\lambda-\sigma)(1+j)(1+o)}$$
$$\times\sum_{k=0}^{n-1}(1+j)^{k+1}w^{n-k-1}v^{k+1}{}_kp_xq_{x+k}\,.$$

4 Special cases

We shall discuss two special cases.

a. $j = o = 0$.

In this case inflation is not included and the interest rate of the credit is not followed. The remaining debt decreases linearly under the amortization plan. We get standard decreasing term insurance which decreases m times per year. This can be noticed in the following way:
The value of Z is defined in the limit case

$$\lim_{w\to 1}\frac{1-w^{(n+1/m-K-S^{(m)})}}{1-w^n} = \frac{1}{n}(n+1/m-K-S^{(m)})\,.$$

So we have

$$Z = \begin{cases} \frac{1}{n}(n+1/m-K-S^{(m)})v^{K+S}\,, & T < n \\ 0\,, & T \geq n \end{cases}$$

and

$$A = \frac{1}{n}\left[\left(n+\frac{1}{m}\right)\frac{i}{\delta}A^1_{x:n} - \frac{i}{\delta}(IA)^1_{x:n} + \frac{i}{\delta}A^1_{x:n} - \frac{i-d^{(m)}}{d^{(m)}\delta}A^1_{x:n}\right],$$

where $A^1_{x:n}$ is a single premium for term insurance and $(IA)^1_{x:m}$ is a single premium for standard increasing term insurance.

b. $i = j$

Let us now consider the situation where the technical interest rate is equal to the inflation rate. We get

$$F_1 = \frac{i^{(m)}}{\delta}\,,$$

$$F_2 = \frac{o}{1+o}\frac{i^{(m)}}{\delta}\frac{1}{m[1-(1+o)^{-1/m}]}\,.$$

Letting $m \to \infty$ we find

$$F_1 = 1$$

$$F_2 = \frac{o}{1+o}\frac{1}{\sigma}\,.$$

5 A deterministic model

For calculations new commutation functions are defined.
We know that

$$v^{k+1}{}_k p_x q_{x+k} = \frac{C_{x+k}}{D_x}\,.$$

In addition we now define new symbols

$$\widehat{F}_1 = \frac{1}{1-w^n}\frac{1}{(1+j)^{1/m}}F_1$$

$$\widehat{F}_2 = \frac{w^{n+1/m}}{1-w^n}\frac{1}{(1+j)^{1/m}}F_2\,,$$

$$\widehat{M}_x = \sum_{k=0}(1+j)^{k+1}C_{x+k}\,,$$

$$\widetilde{M}_x = \sum_{k=0}(1+j)^{k+1}(1+o)^{k+1}C_{x+k}\,.$$

Finally we have

$$A = \widehat{F}\frac{\widehat{M}_x - \widehat{M}_{x+n}}{D_x} - \widehat{F}_2\frac{\widetilde{M}_x - \widetilde{M}_{x+n}}{D_x}\,.$$

6 Example

For a numerical illustration, we assume 1000 units of credit, SLO 54–55 mortality tables, period $m = 12$ and interest rates:

technical interest rate	i	4,5 %
annual inflation rate	j	20 %
credit interest rate	o	12 %

In a table below we compare a single premium for standard decreasing sum insured $j = o = 0$ and a single premium where the future inflation is included.

Standard decreasing term				
	duration			
age	1	2	3	4
22	1,16	2,21	3,24	4,26
30	1,46	2,78	4,10	5,42
37	1,98	3,81	5,64	7,49
45	3,24	6,24	9,29	12,38
50	4,68	9,03	13,46	17,97
60	10,77	20,78	30,94	41,24

Decreasing term with included inflation				
	duration			
age	1	2	3	4
22	1,25	3,58	5,86	8,09
30	1,57	4,52	7,46	10,38
37	2,14	6,22	10,33	14,47
45	3,49	10,22	17,09	24,12
50	5,04	14,80	24,83	35,11
60	11,60	34,05	56,99	80,38

7 Comments

We note that for four-years policy duration we charge double premium, if we take future inflation into account. This result is expected, since the total inflation rate after four years (in our case) is more than 100 %.
Of course it is reasonable to include the inflation rate into a premium only if the following criteria are met:

1. A single premium is charged.
2. The duration is reasonable short.
3. The inflation rate could be predicted (is under control).

4. The inflation rate is more than 10 % per year (otherwise it can be added into technical interest rate).
5. We can not guarantee that the earned interest rate will be high enough to cover inflation plus technical interest rate.
6. Only for term products.

References

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Summary

The paper deals with short term life insurance to guarantee periodic payments related to a credit transaction. In the model the installments and the outstanding debt are regularly adjusted to inflation in the same rhythm as the installments. The inflation is assumed to be constant. The single premium of this insurance is compared with the single premium of a term insurance with a standard decreasing sum.

Zusammenfassung

Untersucht werden Risikoversicherungen zur Deckung der Restschulden von kurzfristigen Krediten. Im betrachteten Modell wird nach jeder Zeitperiode sowohl die zurückzubezahlende Rate als auch die Restschuld entsprechend der Inflation angepasst. Die Inflation wird über die Zeitdauer der Rückzahlungen als konstant angesehen. Die Einmalprämie dieser Versicherung wird mit der Einmalprämie einer linear abnehmenden Restschuldversicherung verglichen.

Résumé

L'articie traite d'assurances temporaires au décès conclues en garantie de paiements périodiques liés à un contrat de crédit. Dans le modèle proposé les paiements restant ainsi que le solde de la dette sont adaptés a l'inflation périodiquement selon le même rythme que les paiements. L'inflation est supposée constante. La prime unique de cette assurance est comparée à celle d'une assurance temporaire au décès décroissant linéairement.