Kurzmitteilungen

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M. Buchwalder, M. Merz, H. Bühlmann,
M.V. Wüthrich, Basel, Zurich

Estimation of Unallocated Loss Adjustment Expenses

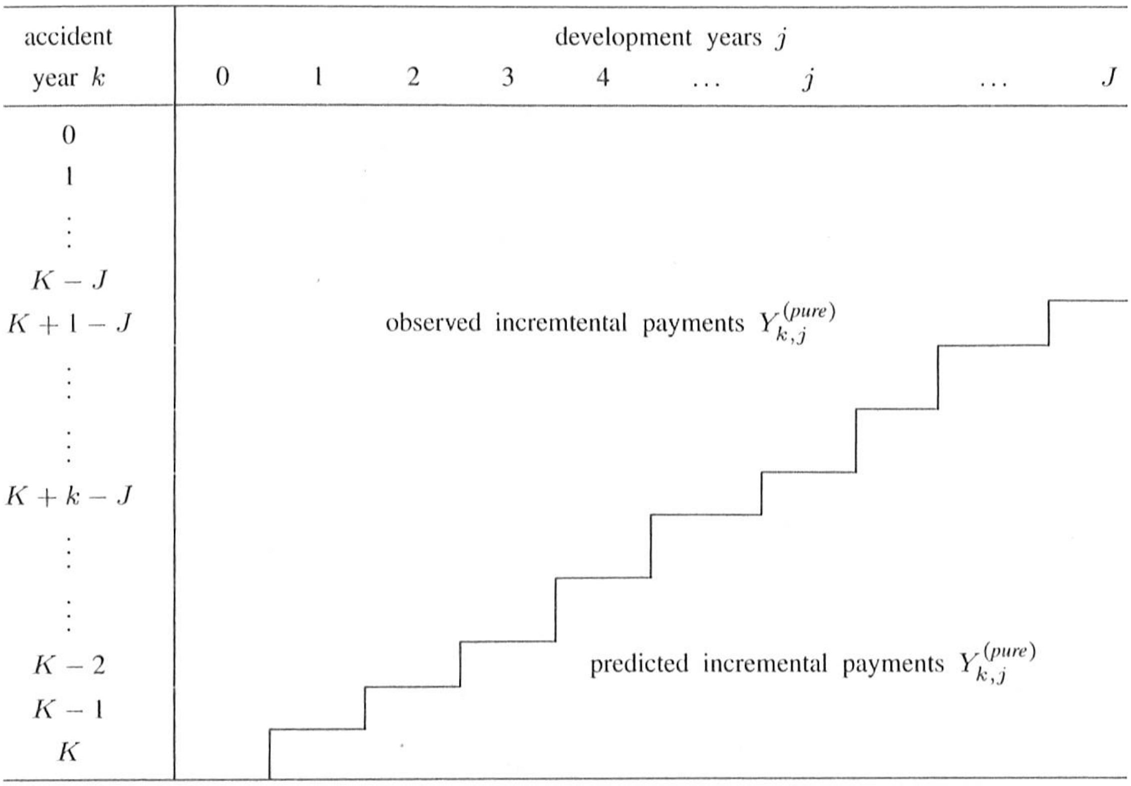
1 Motivation

In this paper we describe the "New York"-method for the estimation of unallocated loss adjustment expenses (ULAE). The "New York"-method can also be found under the name paid-to-paid method. The "New York"-method for estimating ULAE is not new, but it is only poorly documented in the literature (e.g. as footnotes in [2] and [1] or in [4], [3]). Since it is widely used in practice (e.g. also in the Swiss Solvency Test (SST) [6]), we think that it is worth to give a rigorous mathematical approach to this method.
In non-life insurance there are usually two different kinds of claims handling costs, external ones and internal ones. External costs like costs for external lawyers or for an external expertise etc. are usually directly attributable to individual claims and are therefore contained in the usual claims payments and loss development figures. Directly attributable payments are called allocated loss adjustment expenses (ALAE). Typically, internal loss adjustment expenses (claims notification costs, wages of claims handling department, running and maintenance of claims handling system/IT system, management and reporting activities, reinsurance handling, etc.) are not contained in the claims figures and therefore have to be estimated separately. These internal costs related to the settlement of claims can usually not be allocated to single claims. We call costs that are not attributable to individual claims unallocated loss adjustment expenses (ULAE).
In the past, in many accounting principles ULAE were financed in a pay-as-you-go system. In newer accounting principles and also from a solvency point of view, one has also to build reserves for these future costs/expenses for past open claims because they are part of the claims settlement process which guarantees that an insurance company is able to meet all its obligations. In other words, ULAE reserves should guarantee the smooth run off of the old insurance liabilities without "pay-as-you-go" from new business/premium.

2 Notations

2.1 *Pure claims payments*

Usually, claims development figures only consist of "pure" claims payments not containing ULAE charges. They are usually studied in loss development triangles or trapezoids, which have the following structure (we truncate the claims development at J and assume that all claims are settled after development period J) :



We denote by $Y_{k,j}^{(pure)}$ the "pure" incremental payments for accident year k ($0 \leq k \leq K$) in development year j ($0 \leq j \leq J$). "Pure" always means that these quantities contain ALAE but do not contain ULAE. The cumulative pure payments for accident year k after development period j are denoted by

$$X_{k,j}^{(pure)} := \sum_{l=0}^{j} Y_{k,l}{}^{(pure)} . \tag{2.1}$$

We assume that $Y_{k,j}^{(pure)} = 0$ for all $j > J$, i.e. the ultimate pure cumulative loss is given by $X_{k,J}^{(pure)}$.

We have observations for $\mathcal{D}_K := \{X_{k,j}^{(pure)} | \ 0 \leq k \leq K \text{ and } 0 \leq j \leq \min\{J, K-k\}\}$ and the complement of $\mathcal{D}_K$ needs to be predicted.
For the New York-method we also need a second type of development trapezoids, namely a "reporting" trapezoid: For accident year k, $Z_{k,j}^{(pure)}$ denotes the pure cumulative ultimate claim amount for all those claims, which are reported up to (and including) development year j. Hence $\left(Z_{k,0}^{(pure)}, Z_{k,1}^{(pure)}, \dots\right)$ with $Z_{k,J}^{(pure)} = X_{k,J}^{(pure)}$ describes, how the pure ultimate claim $X_{k,J}^{(pure)}$ is reported over time at the insurance company. Of course, this reporting pattern is much more delicate, because sizes which are reported in the upper set $\widetilde{\mathcal{D}}_K := \{Z_{k,j}^{(pure)} | \ 0 \leq k \leq K \text{ and } 0 \leq j \leq \min\{J, K-k\}\}$ are still developing, since usually it takes quite some time between the reporting and the final settlement of a claim. In general, the final value for $Z_{k,j}^{(pure)}$ is only known at the end of year $k + J$.

Remark: Since the New York-method is an algorithm based on deterministic numbers, we assume that all our variables are deterministic. Stochastic variables are replaced by their "best estimate" for its conditional mean at the end of year K. We think that for the current presentation (to explain the New York-method) it is not helpful to work in a stochastic framework.

2.2 *ULAE charges*

The cumulative ULAE payments for accident year k until development period j are denoted by $X_{k,j}^{(ULAE)}$. And finally, the total cumulative payments (pure and ULAE) are denoted by

$$X_{k,j} := X_{k,j}^{(pure)} + X_{k,j}^{(ULAE)}. \tag{2.2}$$

The cumulative ULAE payments $X_{k,j}^{(ULAE)}$ and the incremental ULAE charges

$$Y_{k,j}^{(ULAE)} := X_{k,j}^{(ULAE)} - X_{k,j-1}^{(ULAE)} \tag{2.3}$$

need to be estimated : The main difficulty is that for each accounting year $t \leq K$ we have only one aggregated observation

$$Y_t^{(ULAE)} := \sum_{\substack{k+j=t \\ 0 \leq j \leq J}} Y_{k,j}^{(ULAE)} \qquad \text{(sum over } t\text{-diagonal).} \tag{2.4}$$

In other words, ULAE payments are usually not available for single accident years but rather we have a position "Total ULAE Expenses" for each accounting

year t (in general ULAE charges are contained in the position "Administrative Expenses" in the annual profit-and-loss statement).
Hence, for the estimation of future ULAE payments we need first to define an appropriate model in order to split the aggregated observations $Y_t^{(ULAE)}$ into the different accident years $Y_{k,j}^{(ULAE)}$.

3 New York-method

The New York-method assumes that one part of the ULAE charge is proportional to the claims registration (denote this proportion by $r \in [0,1]$) and the other part is proportional to the settlement (payments) of the claims (proportion $1-r$).

Assumption 3.1 *We assume that there are two development patterns* $(\alpha_j)_{j=0,\dots,J}$ *and* $(\beta_j)_{j=0,\dots,J}$ *with* $\alpha_j \geq 0$, $\beta_j \geq 0$, *for all* j, *and* $\sum_{j=0}^{J} \alpha_j = \sum_{j=0}^{J} \beta_j = 1$ *such that (***cashflow or payout pattern***)*

$$Y_{k,j}^{(pure)} = \alpha_j \cdot X_{k,J}^{(pure)} \tag{3.1}$$

*and (***reporting pattern***)*

$$Z_{k,j}^{(pure)} = \sum_{l=0}^{j} \beta_l \cdot X_{k,J}^{(pure)} \tag{3.2}$$

for all k *and* j.

Remarks :

- Equation (3.1) describes, how the pure ultimate claim $X_{k,J}^{(pure)}$ is paid over time. In fact $(\alpha_j)_j$ gives the cashflow pattern for the pure ultimate claim $X_{k,J}^{(pure)}$. It can easily be seen that this payout model satisfies the classical chain ladder assumptions for cumulative payments (see [5]). Therefore we propose that α_j is estimated by the classical chain ladder factors f_j (see (1) in [5])

$$\widehat{\alpha}_j = \frac{1}{f_j \cdots f_{J-1}} \cdot \left(1 - \frac{1}{f_{j-1}}\right). \tag{3.3}$$

- The estimation of the claims reporting pattern $(\beta_j)_j$ in (3.2) is more delicate. There are not many claims reserving methods which give a reporting pattern $(\beta_j)_j$. Such a pattern can only be obtained if one separates the claims estimates for reported claims and IBNyR claims (incurred but not yet reported).

Model 3.2 *Assume that there exists $r \in [0,1]$ such that the incremental ULAE payments satisfy for all k and all j*

$$Y_{k,j}^{(ULAE)} = \big(r \cdot \beta_j + (1-r) \cdot \alpha_j\big) \cdot X_{k,J}^{(ULAE)}. \tag{3.4}$$

Henceforth, we assume that one part (r) of the ULAE charge is proportional to the reporting pattern (one has loss adjustment expenses at the registration of the claim), and the other part ($1-r$) of the ULAE charge is proportional to the claims settlement (measured by the payout pattern).

Analogously to (2.4) we define the pure claims payments per accounting year

$$Y_t^{(pure)} := \sum_{\substack{k+j=t \\ 0 \le j \le J}} Y_{k,j}^{(pure)} \qquad \text{(sum over } t\text{-diagonal)}. \tag{3.5}$$

Then we are able to define the paid-to-paid ratio :

Definition 3.3 (Paid-to-paid ratio) *We define for all t*

$$\pi_t := \frac{Y_t^{(ULAE)}}{Y_t^{(pure)}} = \frac{\sum_{\substack{k+j=t \\ 0 \le j \le J}} Y_{k,j}^{(ULAE)}}{\sum_{\substack{k+j=t \\ 0 \le j \le J}} Y_{k,j}^{(pure)}} . \tag{3.6}$$

The paid-to-paid ratio measures the ULAE payments relative to the pure claim payments in each accounting year t.

Lemma 3.4 *Assume there exists $\pi > 0$ such that for all accident years k we have*

$$\frac{X_{k,J}^{(ULAE)}}{X_{k,J}^{(pure)}} = \pi . \tag{3.7}$$

Under Assumption 3.1, Model 3.2 and if $X_{k,J}^{(pure)}$ is constant in k, then we have for all accounting years t

$$\pi_t = \pi . \tag{3.8}$$

Proof of Lemma 3.4. We have

$$\pi_t = \frac{\sum\limits_{\substack{k+j=t\\0\le j\le J}} Y_{k,j}^{(ULAE)}}{\sum\limits_{\substack{k+j=t\\0\le j\le J}} Y_{k,j}^{(pure)}} = \frac{\sum\limits_{j=0}^{J} \big(r \cdot \beta_j + (1-r)\cdot \alpha_j\big) \cdot X_{t-j,J}^{(ULAE)}}{\sum\limits_{j=0}^{J} \alpha_j \cdot X_{t-j,J}^{(pure)}}$$

$$= \pi \cdot \frac{\sum\limits_{j=0}^{J} \big(r \cdot \beta_j + (1-r)\cdot \alpha_j\big) \cdot X_{t-j,J}^{(pure)}}{\sum\limits_{j=0}^{J} \alpha_j \cdot X_{t-j,J}^{(pure)}} = \pi\,. \tag{3.9}$$

This finishes the proof. □

We split the claims reserves for accident year k after development period j as follows :

$$R_{k,j}^{(pure)} := \sum_{l>j} Y_{k,l}^{(pure)} = \sum_{l>j} \alpha_l \cdot X_{k,J}^{(pure)} \quad \text{(total pure claims reserves for future paym.)},$$

$$R_{k,j}^{(IBNR)} := \sum_{l>j} \beta_l \cdot X_{k,J}^{(pure)} \quad \text{(IBNyR reserves, incurred but not yet reported)},$$

$$R_{k,j}^{(rep)} := R_{k,j}^{(pure)} - R_{k,j}^{(IBNR)} \quad \text{(reserves for reported claims)}.$$

Result 3.5 (New York-method) *Under the assumptions of Lemma 3.4 we can predict π using the observations π_t (accounting year data, see* (3.8)*). The reserves for ULAE charges for accident year k after development year j, $R_{k,j}^{(ULAE)} := \sum_{l>j} Y_{k,l}^{(ULAE)}$, are given by*

$$\widehat{R}_{k,j}^{(ULAE)} := \pi \cdot r \cdot R_{k,j}^{(IBNR)} + \pi \cdot (1-r) \cdot R_{k,j}^{(pure)}$$
$$= \pi \cdot R_{k,j}^{(IBNR)} + \pi \cdot (1-r) \cdot R_{k,j}^{(rep)}\,. \tag{3.10}$$

Explanation of Result 3.5.

We have under the assumptions of Lemma 3.4 for all k, j

$$
\begin{aligned}
R_{k,j}^{(ULAE)} &= \sum_{l>j} \left(r \cdot \beta_l + (1-r) \cdot \alpha_l\right) \cdot X_{k,J}^{(ULAE)} \\
&= \pi \cdot \sum_{l>j} \left(r \cdot \beta_l + (1-r) \cdot \alpha_l\right) X_{k,J}^{(pure)} \\
&= \pi \cdot r \cdot R_{k,j}^{(IBNR)} + \pi \cdot (1-r) \cdot R_{k,j}^{(pure)} . \qquad (3.11)
\end{aligned}
$$

Remarks :

- In practice one assumes the stationarity condition $\pi_t = \pi$ for all t. This implies that π can be estimated from the accounting data of the annual profit-and-loss statements. Pure claims payments are directly contained in the profit-and-loss statements, whereas ULAE payments are often contained in the administrative expenses. Hence one needs to divide this position into further subpositions (e.g. with the help of an activity-based cost allocation split).

- Result 3.5 gives an easy formula for estimating ULAE reserves. If we are interested into the total ULAE reserves after accounting year t we simply have

$$
\begin{aligned}
\widehat{R}_t^{(ULAE)} &:= \sum_{k+j=t} \widehat{R}_{k,j}^{(ULAE)} \\
&= \pi \cdot \sum_{k+j=t} R_{k,j}^{(IBNR)} + \pi \cdot (1-r) \cdot \sum_{k+j=t} R_{k,j}^{(rep)} , \qquad (3.12)
\end{aligned}
$$

  i.e. all we need to know is, how to split of total pure claims reserves into reserves for IBNyR claims and reserves for reported claims.

- The assumptions for the New York-method are rather restrictive in the sense that the pure cumulative ultimate claim $X_{k,J}^{(pure)}$ must be constant in k (see Lemma 3.4). Otherwise the paid-to-paid ratio π_t for accounting years is not the same as the ratio $X_{k,J}^{(ULAE)}/X_{k,J}^{(pure)}$ even if the latter is assumed to be constant. Of course in practice the assumption of equal pure cumulative ultimate claim is never fulfilled. If we relax this condition we obtain the following lemma.

Lemma 3.6 *Assume there exists $\pi > 0$ such that for all accident years k we have*

$$\frac{X_{k,J}^{(ULAE)}}{X_{k,J}^{(pure)}} = \pi \cdot \left(r \cdot \frac{\overline{\beta}}{\overline{\alpha}} + (1-r) \right)^{-1}, \tag{3.13}$$

with

$$\overline{\alpha} := \frac{\sum_{j=0}^{J} \alpha_j \cdot X_{t-j,J}^{(pure)}}{\sum_{j=0}^{J} X_{t-j,J}^{(pure)}} \quad \text{and} \quad \overline{\beta} := \frac{\sum_{j=0}^{J} \beta_j \cdot X_{t-j,J}^{(pure)}}{\sum_{j=0}^{J} X_{t-j,J}^{(pure)}}. \tag{3.14}$$

Under Assumption 3.1 and Model 3.2 we have for all accounting years t

$$\pi_t = \pi. \tag{3.15}$$

Proof of Lemma 3.6. As in Lemma 3.4 we obtain

$$\begin{aligned} \pi_t &= \pi \cdot \left(r \cdot \frac{\overline{\beta}}{\overline{\alpha}} + (1-r) \right)^{-1} \cdot \frac{\sum_{j=0}^{J} \left(r \cdot \beta_j + (1-r) \cdot \alpha_j \right) \cdot X_{t-j,J}^{(pure)}}{\sum_{j=0}^{J} \alpha_j \cdot X_{t-j,J}^{(pure)}} \\ &= \pi. \end{aligned} \tag{3.16}$$

This finishes the proof. □

Remarks:

- If all pure cumulative ultimates are equal then $\overline{\alpha} = \overline{\beta} = (J+1)^{-1}$ (see Lemma 3.4).
- Assume that there exists a constant $i^{(p)} > 0$ such that for all $k \geq 0$ we have $X_{k+1,J}^{(pure)} = (1 + i^{(p)}) \cdot X_{k,J}^{(pure)}$, i.e. constant growth $i^{(p)}$. If we blindly apply (3.8) of Lemma 3.4 (i.e. we do not apply the correction factor in (3.13)) and estimate the incremental ULAE payments by (3.10) and (3.12)

we obtain

$$\begin{aligned}
\sum_{k+j=t} \widehat{Y}_{k,j}^{(ULAE)} &:= \pi \cdot \sum_{j=0}^{J} \big(r \cdot \beta_j + (1-r) \cdot \alpha_j \big) \cdot X_{t-j,J}^{(pure)} \\
&= \frac{Y_t^{(ULAE)}}{Y_t^{(pure)}} \cdot \sum_{j=0}^{J} \big(r \cdot \beta_j + (1-r) \cdot \alpha_j \big) \cdot X_{t-j,J}^{(pure)} \qquad (3.17) \\
&= \sum_{k+j=t} Y_{k,j}^{(ULAE)} \cdot \left(r \cdot \frac{\overline{\beta}}{\overline{\alpha}} + (1-r) \right) \\
&= \sum_{k+j=t} Y_{k,j}^{(ULAE)} \cdot \left(r \cdot \frac{\sum_{j=0}^{J} \beta_j \cdot \left(1+i^{(p)}\right)^{J-j}}{\sum_{j=0}^{J} \alpha_j \cdot \left(1+i^{(p)}\right)^{J-j}} + (1-r) \right) \\
&\overset{(3.18)}{>} \sum_{k+j=t} Y_{k,j}^{(ULAE)},
\end{aligned}$$

where the last inequality in general holds true for $i^{(p)} > 0$, since usually $(\beta_j)_j$ is more concentrated than $(\alpha_j)_j$, i.e. we usually have $J > 0$ and

$$\sum_{l=0}^{j} \beta_l > \sum_{l=0}^{j} \alpha_l \quad \text{for } j = 0, \ldots, J-1. \tag{3.18}$$

This comes from the fact that the claims are reported before they are paid. Hence if we blindly apply the New York-method for constant positive growth then the ULAE reserves are too high (for constant negative growth we obtain the opposite sign). This implies that without the correction factor in (3.13), we have always a positive loss experience on ULAE reserves in the case of constant positive growth.

- Of course, there are other situations (besides the constant growth model) which satisfy the assumptions of Lemma 3.6. We do not further investigate them, because already the constant growth rate example indicates that one has to be careful about the application of the New York-method.

4 Example

We assume that the observations for π_t are generated by i.i.d. random variables $\frac{Y_t^{(ULAE)}}{Y_t^{(pure)}}$. Hence we can estimate π from this sequence. Assume $\pi = 10\%$.

Moreover $i^{(p)} = 0$ and set $r = 50\%$ (this is the usual choice, also done in the SST [6]). Moreover we assume that we have the following reporting and cash flow patterns ($J = 4$) :

$$(\beta_0, \ldots, \beta_4) = (90\%, 10\%, 0\%, 0\%, 0\%)\,, \tag{4.1}$$

$$(\alpha_0, \ldots, \alpha_4) = (30\%, 20\%, 20\%, 20\%, 10\%)\,. \tag{4.2}$$

Assume that $X_{k,J}^{(pure)} = 1'000$. Then the ULAE reserves for accident year k are given by

$$\left(\widehat{R}_{k,-1}^{(ULAE)}, \ldots, \widehat{R}_{k,3}^{(ULAE)}\right) = (100, 40, 25, 15, 5)\,, \tag{4.3}$$

which implies for the estimated incremental ULAE payments

$$\left(\widehat{Y}_{k,0}^{(ULAE)}, \ldots, \widehat{Y}_{k,4}^{(ULAE)}\right) = (60, 15, 10, 10, 5)\,. \tag{4.4}$$

Hence for the total estimated payments $\widehat{Y}_{k,j} = Y_{k,j}^{(pure)} + \widehat{Y}_{k,j}^{(ULAE)}$ we have

$$\left(\widehat{Y}_{k,0}, \ldots, \widehat{Y}_{k,4}\right) = (360, 215, 210, 210, 105)\,. \tag{4.5}$$

We consider now the following claims development trapezoid for the pure incremental payments $Y_{k,j}^{(pure)}$ with $J = 4$ and $K = 6$:

	0	1	2	3	4
0	300	200	200	200	100
1	300	200	200	200	100
2	300	200	200	200	100
3	300	200	200	200	
4	300	200	200		
5	300	200			
6	300				

Hence in the run-off situation we have for the accounting year payments under our cashflow assumptions :

$$\left(Y_7^{(pure)}, \ldots, Y_{10}^{(pure)}\right) = (700, 500, 300, 100)\,, \tag{4.6}$$

$$\left(Y_7^{(ULAE)}, \ldots, Y_{10}^{(ULAE)}\right) = (40, 25, 15, 5)\,, \tag{4.7}$$

Therefore as soon as we have "experienced" the full β-pattern, the ULAE payments are simply $r \cdot \pi = 5\%$ of the pure payments.

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Mario V. Wüthrich
ETH Zürich
Department of Mathematics
CH-8092 Zürich
Switzerland

email : mario.wuethrich@math.ethz.ch