Developments in nonlinear fracture mechanics

*Angewandte Mechanik in Industrie und Hochschule IV**

Developments in Nonlinear Fracture Mechanics

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Es wird eine Übersicht über den Stand der Technik in Bruchmechanik, unter spezieller Berücksichtigung neuer Entwicklungen in Bruchmechanik von Stahlkonstruktionen grosser Zähigkeit, gegeben.

Die Gebiete der dynamischen Rissausbreitung und Risshemmung werden ebenfalls dargelegt und mögliche Anwendungen dieser Theorien erörtert. Einige Anwendungen der Bruchmechanik auf Strukturen wie Druckkessel, Flugzeuge, Schiffe und Grosskraftwerksanlagen werden kurz vorgestellt.

A review is given of the state of the art of fracture mechanics with special attention to recent developments of non-linear fracture mechanics as developed for high ductility structural steels.

The areas of dynamic crack propagation and crack arrest are also reviewed and possible applications of these theories discussed. Some applications of fracture mechanics to structures such as pressure vessels, airplanes, ships and heavy power energy equipment is briefly presented.

Fracture Mechanics is a young science. It has evolved during the last three decades. Its theoretical foundation falls back on *Griffiths* isolated work around 1920 and his energy criterion for crack extension.

The linear elastic fracture mechanics (LEFM) is today well established and has a strong position. It has proved to be successful in treating crack growth in connection with fatigue and brittle fracture. The relevant parameter in LEFM the stress intensity factor K_I can be obtained from a linear elastic analysis. Therefore LEFM has profited strongly from the *numerical finite element methods* developed for structural analysis.

LEFM has however strong limitations in its application to structural materials and is not valid for modern structural steels of normal dimensions. Since about ten years intensive work is going on developing methods for treatment of fracture problems in ductile materials—elastic-plastic or nonlinear fracture mechanics (EPFM).

This work has partly been very fundamental and resulted in solutions of elastic-plastic crack problems. It has also aimed at development of experimental methods for material testing and of engineering methods for failure analysis. This work will be reviewed in this paper.

Solutions of elastic-plastic crack problems

For a material with an exponential stress strain relation $\varepsilon/\varepsilon_0 = \alpha(\sigma/\sigma_0)^n$ the stresses and strains at a crack-tip are of the form [1, 2].

$$(1) \quad \sigma \sim \sigma_0 (J/\sigma_0 r)^{\frac{1}{n+1}}, \qquad \varepsilon \sim \varepsilon_0 (J/\sigma_0 r)^{\frac{n}{n+1}}$$

where r is distance from the crack tip. J is the wellknown *J-integral* [3]. It may be determined by integration of stress and strain quantities along an arbitrary path around the crack tip or from the variation of the elastic potential U with respect to crack length.

$$(2) \quad J = \int_0 (W dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds) = -\frac{\partial U}{\partial a}$$

Thus J may be calculated from quantities which are determined in a structural analysis.

Criterion for initiation of crack growth

From Eq. (1) it is seen that the "amplitude" of the singularity in a non-linear material is governed by J. Under certain conditions J may also be interpreted as the rate of energy flow to the crack tip during crack extension. This indicates that J is a suitable parameter in a criterion for initiation of crack growth. It has also been shown in several experimental investigations e.g. [4, 5], that the criterion

$$(3) \quad J_I = J_{Ic}$$

well predicts initiation of crack growth in ductile materials. J_{Ic} is a material parameter the fracture thoughness. Experimental methods to determine J_{Ic} are today established and requires in principle only one specimen for each discrete temperature.

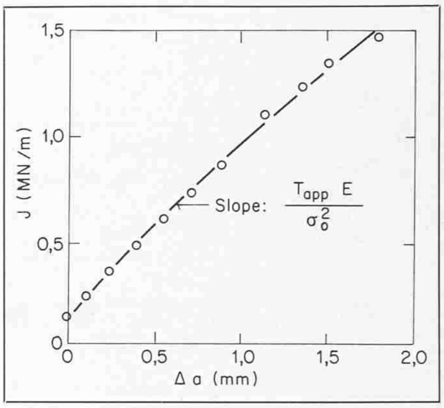


Fig. 1. Experimental points and fitted curve for $J_R = J_R(\Delta a)$

However, initiation is not as critical an event in a ductile material as in a more brittle one. In ductile materials one has a phase of stable growth between initiation of crack growth and unstable growth.

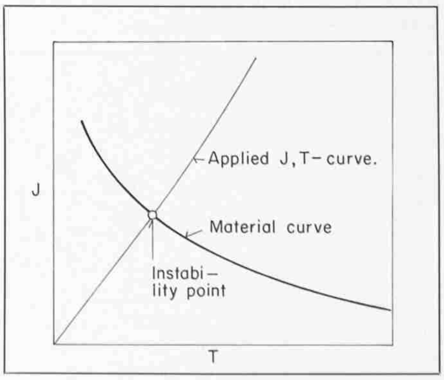
Criteria for stable growth and instability

For many materials J increases approximately linearly with crack growth Δa during the phase of stable growth, Fig. 1. The curve $J = J_R(\Delta a)$ is by hypothesis assumed to be a material curve independent of structural shape and only a function of Δa. The non-dimensionalized slope of the J_R-curve $T_{mat} = E_0 \sigma_0^{-2} dJ_R/da$ called the *tearing modulus* was introduced by *Paris* [6] as a measure of the materials resistance to stable crack growth.

For any structure containing a crack J may be determined as a function of load and crack length, Fig. 1. From this J an applied tearing modulus $T_{appl} = E_0 \sigma_0^{-2} dJ/da$ may be determined.

The conditions for stable growth is $J =$

Fig. 2. Material-curve and loading curve represented in $J = J(T)$ diagram



* Vgl. Schweizer Ingenieur und Architekt, Heft 51/52: 1117-1121, 1982; Heft 1/2: 2-7, Heft 4: 42-46, 47-50; Heft 9: 275-278, 1983.

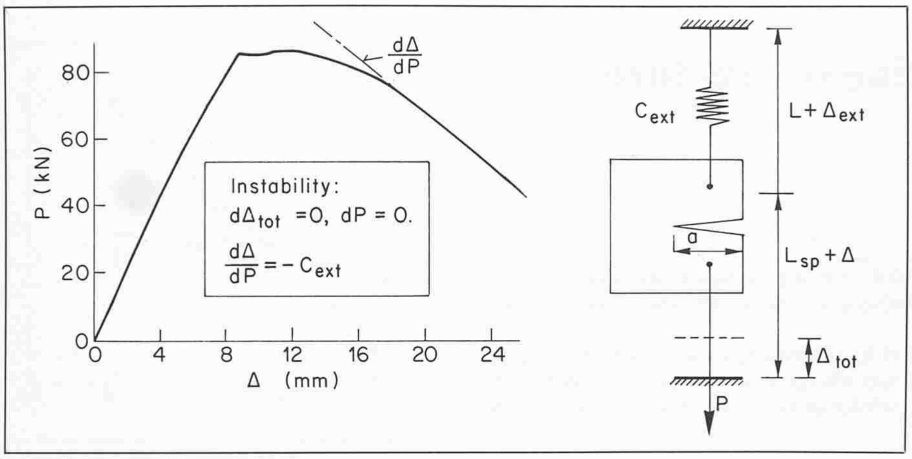


Fig. 3. Instability criterion for nonlinear component in series with elastic structure is $d\Delta/dP = -C_{ext}$

J_R and $T_{appl} < T_{mat}$. Instability occurs when

(4) $T_{appl} \geqq T_{mat}$

For small amounts of crack growth T_{mat} is often constant and the above analysis is simple to perform analytically. For larger amounts of crack growth T_{mat} often varies with Δa. It is then common to present material properties and the loading curve in a T–J-diagram, Fig. 2.

To treat instability of a system the whole system has to be analysed, not only the failing component. This is demonstrated by the following simple example.

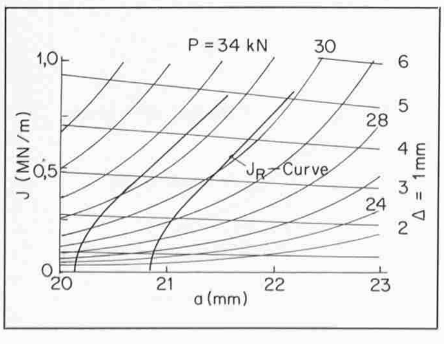
The load displacement relation for a component with crack growth and plastic deformation e.g. a CT specimen looks as shown in Fig. 3. If this component is put in series with an elastic, external component with compliance $\Delta/P = C_{ext}$ instability of the whole system occurs when

(5) $$-\frac{dP}{d\Delta} = \frac{1}{C_{ext}}$$

For crack problems in structures where the non-linear, cracked component can be identifyed and tested separately it is convenient and reliable to use eq. (5) as an instability criterion [7] instead of eq. (4).

As shown in [7], the conditions Eq. (4) also satisfies condition Eq. (5). However the T-criterion is assumed to be geometry independent whereas eq. (5) certainly is not.

Fig. 4. Crack driving force diagram for bend specimen. Curves of J for P = const., Δ = const. and material J_R-curve



An alternative method to treat instability problems has been worked out by *Shih* and *Hutchinson* [8]. The method results in the so called *"crack driving force" diagram*. It is based on linear-elastic and exponentially hardening material models. In the crack force diagram the material J_R curve is introduced, together with parametric curves for $J = J(P, \Delta)$, Fig. 4.

Instability occurs when $(dJ/da)_{\Delta_t} > dJ_R/da$ which is equivalent to Eq. (4). The instability point may be determined from the diagram. A handbook has been worked out to treat crack problems with this procedure, the EPRI Handbook, [9].

A third method for treating nonlinear problems is the so called *"failure assessment"* or *R6-diagram*. Originally it was based on a linear and a limit load analysis and an interpolation between these. Today the failure assessment diagram is usually based on a general non-linear analysis of the type discussed above. The procedure is complicated to use for other cases than load control.

Methods of analysis

An advantage of the J-based fracture mechanics is that it is well adapted to the normal methods of structural analysis. Especially the relation of J to the potential U is useful and leads to simple interpretation of J and corresponding simple formulas to evaluate J from load-displacement diagrams.

For the three point bend specimen J becomes simply:

(6) $$J = \frac{2W}{b}$$

with W being strain energy per unit specimen thickness and b ligament width.

For a bend specimen in series with an external spring one has for the T-modulus

(7) $$T = \frac{E}{\sigma_0^2}\frac{\partial J}{\partial a} = \frac{4\,EP^2}{\sigma_0^2 b^2}\left(\frac{1}{C} + \frac{\partial P}{\partial \Delta_c}\bigg|_a\right)^{-1} - \frac{J}{b}$$

Here C is the compliance of the external elastic structure in series with the specimen including the uncracked specimen compliance. Similar expressions can be derived for other structures. Determination of J and T for a general structure with cracks of general e.g. elliptic form is however very difficult to make and also expensive since time-consuming finite element calculations are required.

Experimental methods

The material testing required to determine the material properties J_{Ic} and T_{mat} is essentially the same as for K_{Ic}. One significant difference is however the need to measure crack extension more accurately. Crack length changes has to be measured with an accuracy of 0.1 mm during stable crack growth. Several methods are used to achieve this, e.g. electrical methods based on direct current resistance, high frequency impedance, eddy current phenomena etc and compliance methods based on the change of compliance with crack length.

Application

The development of EPFM has been stimulated and supported by the *nuclear reactor industry* and its *safety problems*, e.g. the risk of failure in piping systems and in pressure vessels.

For unstable fracture to occur in a piping system the compliance of the pipe i.e. its length is of fundamental importance as is evident from Eqs. (4) and (7). The larger the compliance i.e. the longer the pipe the greater is the risk of instability during crack growth.

An approximate analysis of a rigidly fastened, long pipe with a circumferential crack exposed to limit load bending moment gives the following expression for T_{appl} [10].

(8) $$T_{appl} = F_1(\Theta)\,\frac{L}{R}$$

where L = pipe length, R pipe radius, 2 θ angular extension of the crack and F_1 (θ) a function of θ. For $0 < 2\,\theta < 120°$ one has $0{,}2 > F_1 > 1{,}2$. This shows that

T_{appl} mainly depends on pipe flexibility i.e. length L. For stainless steel $T_{mat} > 200$ and thus with $R = 0{,}25$ m a length $L > 50$ m is required for Eq. (4) to be satisfied and instability to occur even for cracks extending over one third of the circumference ($2\,\theta = 120°$)

Another example of the compliance sensitivity of instability offers the wide plate with edge cracks (DEC) or a center crack (CC). When limit load is reached one has [6].

$$(9) \qquad T_{appl} = 2\alpha \cdot t\,EC = 2\,\alpha\,L/W$$

where $C = L/WtE$ is compliance, L length, W width, t thickness and $\alpha = 1$ for CC and 4/3 for DEC.

As is seen T_{appl} and thus the tendency for instability increases with the length L.

Discussion

The elastic-plastic fracture mechanics is today a valuable tool in fracture analysis. However many problems remain to be solved before it can be easily applied to general three-dimensional problems of stable crack growth and failure. A certain geometry dependance has been observed for the T modulus. This gives some uncertainty concerning its generality. Actually neither the J integral nor the T-modulus are theoretically well founded for application to stably growing cracks.

In spite of this the concept have proved to be efficient in treating failure problems in ductile materials. An intersting application of the T-modulus which is elaborated today is crack growth due to low cycle fatigue. Here the T concept seems to be powerful [11] in predicting crack growth rate and life to failure beyond the regim where Paris law is valid.

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Niederländischer Betonstrassentag in Utrecht

Von G. Brux, Frankfurt a. Main

Unter den über 800 Teilnehmern am Niederländischen Betonstrassentag (19. November 1982) in Utrecht befanden sich Fachleute aus Forschung, Lehre und Praxis aus Belgien, der Bundesrepublik Deutschland und Grossbritannien. *A.S.G. Bruggeling*, Professor an der Technischen Hochschule Delft und Vorsitzender des Niederländischen Betonvereins (Betonvereniging, BV) eröffnete die zusammen mit der Forschungsanstalt für Strassenbau (SCW) und dem Verein der Niederländischen Zementindustrie (VNC) ausgerichtete Fachtagung.

Betonstrassenbau in den 80er Jahren

Nach *J.C. Slagter* (Verkehrsministerium, Den Haag) haben neuere Entwicklungen und die steigenden Strassenbelastungen in den letzten Jahren in *Belgien*, der *Bundesrepublik Deutschland*, *Großbritannien* und den *USA* zu *vermehrtem Bau von Autobahnen mit Betondecken* geführt. In den Niederlanden wurde lediglich 1977 die 1961 mit Asphaltdecke (10 cm) auf Magerbeton (20 cm) erbaute Autobahn AB 28 auf 2,5 km und nur auf einer Fahrbahnseite mit einer durchgehend bewehrten Betondecke (16 cm) und auf 1,5 km mit einer unbewehrten Betondecke (21 cm) mit verdübelten Fugen überdeckt und ein Jahr später zum Vergleich dazu ein anschliessendes Teilstück mit Asphaltdecke (4 cm auf 4 oder 11 cm Ausgleichsschicht) überzogen. Um eigene Erfahrungen im Betonstrassenbau zu sammeln, soll ein Abschnitt der im Bau befindlichen Autobahn E 8 eine unbewehrte Betondecke (20-22 cm) mit verdübelten Fugen im Abstand von 5 m auf einer Magerbetonschicht (20 cm) erhalten. Die derzeitig ungünstige Wirtschaftslage zwingt zu Abstrichen im Strassenbau, was sich auch auf den Betonstrassenbau auswirkt.

Bemessung von Betondecken für Strassen

Nach *H.E. van der Most* (Betonforschungsstelle der Niederländischen Zementindustrie [BNC], s'Hertogenbosch), Vorsitzender der SCW-Arbeitsgruppe C 1, werden beim beschriebenen BNC-Verfahren die Temperaturspannungen nach *Eisenmann* (BRD) und die Verkehrsspannungen nach *Westergaard*

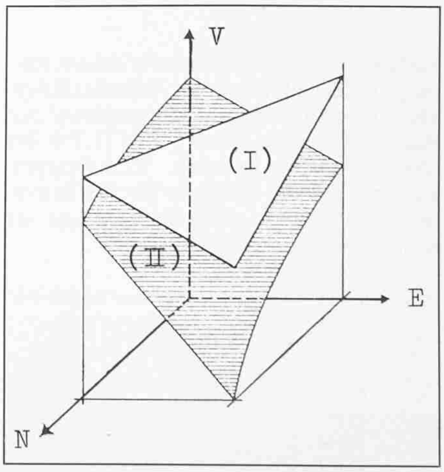

Bild 1. Fahrkomfort von Betonstrassen mit Dübel (I) und ohne Dübel (II) in Abhängigkeit von der Belastung durch Verkehr (V), von Einflüssen der Natur N und vom Erosionswiderstand E des Unterbaus

(USA) berücksichtigt; die Betondecke wird nicht nur auf Festigkeit, sondern auch auf Biegesteifigkeit im Fugenbereich bemessen, was durch Untersuchungen von *Packard* (USA) bestätigt wurde und damit bessere Befahrbarkeit erreicht. Die Biegesteifigkeit lässt sich durch eine grössere Plattendicke, besseren und steiferen Unterbau, Verzahnung und Dübel vergrössern. Geschildert